# Nitsche-based overlapping and fictitious domain methods for the Stokes problem

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Overlapping and fictitious domain methods offer many advantages over standard finite element methods that require the generation of a single high-quality mesh conforming to computational domain. For instance, with overlapping mesh methods, the computational domain may instead be decomposed into different subdomains which then may be meshed independently. Fictitious domain methods, on the other hand, allow to embed complex geometries only described by their boundary surfaces into a structured background mesh. Both approaches require a special treatment of the imposed boundary and interface conditions.

Nitsche's method provides a general approach to weakly imposing boundary and interface conditions in a Lagrange multiplier-free way. Recently, the ideas behind Nitsche's method have been extended to propose overlapping [1] and fictitious domains [2] formulations. Here, a main challenge is to design the methods to be insensitive with regard to the interface and boundary position.

In this work, we present Nitsche-based formulations for a class of stabilized finite element methods for the Stokes problem posed on fictitious [3] and overlapping domains [4]. We address various ways to make the formulation robust and to avoid ill-conditioned linear algebra systems by adding certain so-called ghost-penalties in the vicinity of the boundary and interface. As a consequence, the resulting methods are inf-sup stable and optimal order *a priori* error estimates can be established. Moreover, the condition number of the resulting stiffness matrix is shown to be bounded independently of the location of the boundary and interface. Finally, we present numerical examples in three spatial dimensions confirming the theoretical results and illustrating the applicability of the methods to complex 3D geometries.

References:

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