A fully automatic hp-adaptive refinement strategy for Maxwell's equations

Markus Bürg[1]

The performance of the classical finite element method can be improved by mesh refinement (h-refinement) or the use of higher order ansatz spaces (p-refinement). A combination of both (hp-refinement) can lead to exponential convergence of the computed solution.

Nowadays adaptive refinement of the computational domain is a widely used feature to obtain an accurate numerical solution of the partial differential equation with as less computational work as possible. Therefore one needs to decide, where the approximation error of the numerical solution is relatively large and, thus, refinement should take place. Since the analytic solution is ussually not known, one has to estimate the approximation error in terms of the numerical solution to be able to decide, which areas of the computational domain have to refined further.

In recent years a broad interest in the numerical solution of Maxwell's equations has come up, because this system of equations appears in a lot of nano-scaled processes due to the presence of an electromagnetic field. Solving these equations numerically usually requires a lot of computational work and a fully hp-adaptive refinement strategy can reduce the amout of work, which is related to solving the stationary problems, significantly. We present an hp-efficient residual-based a posteriori error estimator for Maxwell's equations in the electric field formulation, which gives a reliable and robust estimation of the true energy error. Then an hp-refinement strategy is introduced, which is based on the solution of local boundary value problems.

These are the major ingredients for a fully automatic hp-adaptive refinement algorithm, which creates a sequence of problem-adapted finite element approximation spaces allowing the convergence of the finite element solution towards the analytic solution with an exponential rate of convergence.

[1] KIT, Institut für Angewandte und Numerische Mathematik 2, Karlsruhe, Germany, `buerg@kit.edu`