Condition number estimates for the finite element method on adaptive meshes

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It has been demonstrated that significant improvements in accuracy can be gained when an appropriately chosen anisotropic mesh is used in the numerical solution of PDEs exhibiting anisotropic features. However, an anisotropic mesh could contain elements of large aspect ratio and very small volume and, consequently, there exists a concern in the numerical analysis community that an adaptive (anisotropic) mesh can lead to linear algebraic systems which are ill-conditioned and difficult to solve and this may outweigh the accuracy improvements gained if using an anisotropic mesh.

The objective of this work is to study the condition number of linear systems resulting from linear FEM discretization on adaptive meshes. We use the technique introduced by I. Fried and compute bounds on the condition number of the stiffness matrix for arbitrary meshes with the help of an appropriately chosen fictitious density distribution. The estimates involve eigenvalues of the mass matrix and the principal eigenvalue of the structure.

A surprising result is that the condition number on adaptive meshes is much better than generally assumed, especially in two dimensions.

We further develop a simple diagonal scaling which employs available mesh information and significantly improves the condition number of the resulting linear system.

References:

[1] I. Fried, *Bounds on the spectral and maximum norms of the finite element stiffness, flexibility and mass matrices*, Int. J. Solids Struct., 9(9):1013–1034, 1973.

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