FAKULTÄT FÜR
MATHEMATIK

Chemnitz FEM-Symposium 2011

Programme

Collection of abstracts

List of participants

TECHNISCHE UNIVERSITÄT CHEMNITZ

1836-2011
175 Jahre

Chemnitz, September 28 - 30, 2011

TECHNISCHE UNIVERSITÄT CHEMNITZ

Fakultät für Mathematik

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Scientific topics:

The symposium is devoted to all aspects of finite elements and wavelet methods in partial differential equations.

The topics include (but are not limited to)

- adaptive methods,
- parallel implementation,
- high order methods.

This year we particularly encourage talks on

- hp-FEM and Applications
- Variational Inequalities
- Saddle Point Problems

Invited Speakers:

Alexander Düster (Leibniz Universität Hannover)

Ernst P. Stephan (Technische Universität Hamburg-Harburg)

Walter Zulehner (Johannes Kepler Universität Linz)

Scientific Committee:

Th. Apel (München), S. Beuchler (Bonn), G. Haase (Graz),
H. Harbrecht (Basel), R. Herzog (Chemnitz), M. Jung (Dresden),
U. Langer (Linz), A. Meyer (Chemnitz), A. Rösch (Duisburg),
O. Steinbach (Graz)

Organising Committee:

J. Rückert, A. Günnel, H. Schmidt, R. Schneider, M. Pester, K. Seidel, A.-K. Glanzberg

WWW: `http://www.tu-chemnitz.de/mathematik/fem-symposium/`

Internet access:

A laptop with internet connection is available near the conference rooms.

username: "fem11"
password: "fem11"

The hotel provides an open wireless network for free, accessible in most of the rooms, but at least in the lounge.

Food:

The conference fee includes:

- Lunch on all three days of the symposium (one soft drink is included)
  Further drinks are on your own expense.
- The conference dinner on Wednesday.
- The excursion on Thursday.
- Tea, coffee, soft drinks and snacks during breaks.

Dinner on the other days is not included.

Recreation:

The hotel offers sauna for free (past 4 p.m., contact reception).

Programme for Wednesday, September 28, 2011

9:00 **Opening**
A. Meyer
Room: "Linde"

Variational Inequalities
Chairman: A. Meyer
Room: "Linde"

11:10 *Tea and coffee break*

FEM on Manifolds / Parallel Algorithms
Chairman: G. Of
Room: "Linde"

12:30 *Conference Photo*

12:40 *Lunch*

Saddle Point Problems
Chairman: O. Steinbach
Room: "Linde"

16:05 *Tea and coffee break*

	Optimal Control *Chairman*: T. Apel *Room*: "Linde"	**Convection Diffusion** *Chairman*: O.Sander *Room*: "Lärche"
16:35	R. Herzog . . . . 19 A priori error estimates for an elliptic control problem with non-differentiable cost functional.	N. Ahmed . . . . 24 Stabilised finite element discretisations applied to an operator splitting method of population balance equations.
17:00	L. John . . . . 20 Optimal Dirichlet boundary control for the Navier–Stokes equations.	C. Hofreither . . . . 25 A non-standard finite element method for convection-diffusion-reaction problems on polyhedral meshes.
17:25	J. Pfefferer . . . . 21 Neumann boundary control of semilinear elliptic equations: finite element discretization and error estimates.	T. Linß . . . . 22 Maximum-norm a posteriori error estimators for parabolic problems.
17:50	G. Wachsmuth . . . . 23 Optimal control of quasistatic plasticity.	M. Schopf . . . . 26 Convergence and stability in balanced norms of finite element methods on Shishkin meshes for reaction-diffusion problems.
19:00	*Conference Dinner*	

Programme for Thursday, September 29, 2011

***hp* Methods**
Chairman: M. Jung
Room: “Linde”

9:00	A. Düster The finite cell method: a fictitious domain approach applying high-order finite elements.	27
9:50	S. Beuchler Schwarz type solvers for *hp*-FEM discretizations of mixed problems.	28
10:15	M. Bürg A fully automatic *hp*-adaptive refinement strategy for Maxwell’s equations.	29
10:40	A. Dedner Generic construction of high order finite-element spaces.	30
11:05	*Tea and coffee break*	

***hp* Methods**
Chairman: S. Beuchler
Room: “Linde”

11:30	W. Medjroubi Frequency selection in heaving airfoil wakes using a high-order numerical method.	31
11:55	W. Mitchell Comparison of *hp*-adaptive finite element strategies.	32
12:20	G. Matthies Higher order variational time discretizations for nonlinear systems of ordinary differential equations.	33
12:45	A. Springer Efficient numerical realization of discontinuous Galerkin methods for temporal discretization of parabolic equations.	34
13:10	*Lunch*	
14:30	*Excursion*	
19:00	*Meeting of the Scientific Committee*	

Programme for Friday, September 30, 2011

	Adaptivity *Chairman*: A. Rösch *Room*: "Linde"	**Mechanics** *Chairman*: R. Herzog *Room*: "Lärche"
9:00	A. Große-Wöhrmann . . . . 35 A posteriori control of modeling and discretization errors in finite elastoplasticity.	H. Schmidt . . . . 40 Axisymmetric problems in continuum mechanics with transversely isotropic materials.
9:25	L. Kamenski . . . . 36 Condition number estimates for the finite element method on adaptive meshes.	J. Rückert . . . . 41 Basic modelling for large deformation on plates.
9:50	M. Neumüller . . . . 37 An adaptive DG finite element method in the space-time domain.	A. Günnel . . . . 42 A plate equation for elasticity with large deformations.
10:15	M. Weise . . . . 38 Simulation of fibre reinforced polymers with adaptive FEM.	P. Steinhorst . . . . 43 FEM-simulation of crack propagation in linear piezoelectric material.
10:40	H. Egger . . . . 39 Mixed finite element methods for radiative transport.	Y. Zhu . . . . 44 Multiphase field modeling chemical vapor infiltration.
11:05	*Tea and coffee break*	
11:30	**Parallel Algorithms** T. Apel *Room*: "Linde"	
11:30	G. Haase . . . . 45 Parallel acceleration: achievements and pitfalls.	
11:55	A. Meyer Closing	
	Lunch	

Some finite element approaches for contact/obstacle problems

Ernst P. Stephan[1]

The first part of the talk deals with dual formulations for unilateral contact problems with Coulomb friction. Starting from the complementary energy minimization problem, Lagrangian multipliers are introduced to include the governing equation, the symmetry of the stress tensor as well as the boundary conditions on the Neumann and contact boundary. Since the functional arising from the friction part is nondifferentiable an additional Lagrangian multiplier is introduced. This procedure yields a dual-dual formulation of a two-fold saddle point structure. Two different Inf-Sup conditions are introduced to ensure existence of a solution. The system is solved with a nested Uzawa algorithm.

In the second part of the talk a mixed hp-time discontinuous Galerkin method for elasto-dynamic contact problem with friction is considered. The contact conditions are resolved by a biorthogonal Lagrange multiplier and are component-wise decoupled. On the one hand the arising problem can be solved by an Uzawa algorithm in conjunction with a block-diagonalization of the global system matrix. On the other hand the decoupled contact conditions can be represented by the problem of finding the root of a non-linear complementary function. This non-linear problem can in turn be solved efficiently by a semi-smooth Newton method. The second method can also be applied to parabolic obstacle problems, e.g. pricing American put options.

In all cases numerical experiments are given demonstrating the strengths and limitations of the approaches.

The talk is based on a joint work with M. Andres and L. Banz.

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On approximation of higher order for variational inequalities of mixed type

Joachim Gwinner[1]

In this talk we are concerned with the finite element method in its p-version to treat a scalar variational inequality of the mixed type that models unilateral contact and Coulomb friction or other nonsmooth material behaviour in continuum mechanics.

This leads to a nonconforming discretization scheme. In contrast to previous work we employ Gauss-Lobatto quadrature for the approximation of the unilateral constraint and also for the friction-type functional. We take the resulting quadrature error into account of the error analysis.

At first without any regularity assumptions, we prove convergence of the FEM Galerkin solution in the energy norm. To this end we investigate Mosco-Stummel-Glowinski convergence [3] for both convex sets and convex functionals. The key of our norm convergence result for the $p-$FEM is the used Gauss-Lobatto integration rule with its high exactness order and its positive weights together with duality arguments in the sense of convex analysis.

Secondly by a novel Céa-Falk lemma we split the total discretization error into two different parts: the distance of the continuous solution to the convex set of approximations in the trial space and the consistency error caused by the nonconforming approximation. Here we use the well-known approximation theory of spectral methods [1], the cutting technique of Falk [2], and interpolation arguments. Thus we arrive under mild regularity assumptions at an a priori error estimate which is suboptimal because of the treatment of the consistency error in the nonconforming approximation scheme and because of the regularity threshold in unilateral problems.

References:

[1] C. Bernardi, Y. Maday, Spectral Methods, in: P.G. Ciarlet and J.L. Lions, eds. *Handbook of Numerical Analysis V, part 2* (Elsevier, Amsterdam, 1997) 209 – 485.
[2] R.S. Falk, Error estimates for the approximation of a class of variational inequalities, Math. Comp. 28 (1974) 963 – 971.
[3] R. Glowinski, *Numerical methods for nonlinear variational problems,* (Springer, New York, 1984).

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Reduced basis methods for parametrized variational inequalities

Bernard Haasdonk[1] Julien Salomon[2] Barbara Wohlmuth[3]

Reduced Basis Methods are increasingly popular methods for simulation-based model reduction of parametrized problems. Different types of partial differential equations have been treated, in particular linear and nonlinear, coercive and non-coercive stationary problems, as well as parabolic and hyperbolic evolution problems. Systems can also be dealt with, such as Stokes, Navier-Stokes and Boussinesq equations, etc. see [1] and references therein. Key ingredients are snapshot-based reduced basis space construction with (POD)-greedy techniques [2], rigorous a-posteriori error control by residual analysis, and rapid online simulations by decoupling into an offline and online simulation phase. In the current presentation, we address a new class of problems, which are given by combining partial differential equations with inequality constraints [3]. These can be obtained from constrained variational problems, such as occurring in contact problems. We give a formulation of a reduced basis method for parametrized variational inequalities. We explain special considerations in reduced basis construction for primal and dual variables, and comment on the computational procedure for the reduced problem. The reduced problem allows a full offline/online decomposition, which enables rapid online simulations independent of the dimension of the full problem. We give several analytical results for the reduced solutions such as well-posedness, boundedness by the data functions and Lipschitz-continuity with respect to the parameters. A-posteriori error estimation is more difficult than in the pure equality context, still it is possible to derive rigorous error bounds. The a-posteriori error estimators can for instance be used in the offline-phase for reduced basis generation with greedy procedures. Numerical examples demonstrate different aspects of approximation quality and computational time of the reduced scheme and the quality of the error estimators.

References:

[1] G. Rozza, D.B.P. Huynh, and A.T. Patera, "Reduced basis approximation and a posteriori error estimation for affinely parametrized elliptic coercive partial differential equations: application to transport and continuum mechanics." *Arch. Comput. Meth. Eng.*, 15(3):229–275, 2008.
[2] B. Haasdonk, M. Ohlberger, "Reduced Basis Method for Finite Volume Approximations of Parametrized Linear Evolution Equations." *M2AN, Math. Model. Numer. Anal.*, 42(2):277-302, 2008.
[3] B. Haasdonk, J. Salomon, and B. Wohlmuth, "A Reduced Basis Method for Variational Inequalities", *SimTech Preprint 2011-17, University of Stuttgart*, 2011.

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Boundary element methods for variational inequalities

Olaf Steinbach[1]

Variational inequalities to be considered in $H^{1/2}(\Gamma)$ result from different applications, e.g. boundary value problems with boundary conditions of Signorini type, contact problems in elasticity, or from constrained optimal control problems. In this talk we will discuss both the error analysis of the boundary element solution, and suitable iterative solution strategies.

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Geodesic finite elements

Oliver Sander[1]

Geodesic finite elements are a novel way to discretize problems involving functions with values on a Riemannian manifold. Examples for such problems include Cosserat materials and liquid crystals. Geodesic finite elements are conforming and invariant under isometries of the value manifold. Numerical evidence suggests that the discretization error rate behaves optimally. We present the theory of geodesic finite elements and give a few example applications.

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FETI-method for biomechanical applications

Christoph Augustin[1] Olaf Steinbach[2]

In this talk the focus will be on the structural model for the nonlinear elastic behavior of biological tissues, for example arterial walls or cardiac tissue. These materials are anisotropic due to a preferential orientation of collagen fibers in the tissue and consist of several layers. The resulting nonlinear models lead to very complex and time-consuming algorithms.

A way to treat these algorithms is the strategy of parallel computing. One possibility to achieve such a parallelization is to apply domain decomposition methods, which are also motivated by the composition in layers of the considered biological tissues. We outline the main ideas of one particular approach, the finite element tearing and interconnecting (FETI) method and its application to biomechanical models.

Finally numerical examples are included where we compare different solution and preconditioning techniques.

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Efficient solvers for saddle point problems with applications to PDE-constrained optimization

Walter Zulehner[1]

In this talk we present a general framework for analyzing saddle point problems. This includes a discussion of sharp estimates for the solution of such problems in terms of the data. We also study possible strategies for constructing norms for parameter-dependent problems leading to robust (i.e.: parameter-independent) estimates.

On the discrete level this framework can be used to construct and analyze preconditioners for the discretized optimality system of PDE-constrained optimization problems. We focus on preconditioners which are based on multigrid techniques.

For the several classes of distributed optimal control problems we discuss the efficiency of the described methods.

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Incompressible, elastic materials and large deformations

Martina Balg[1] Arnd Meyer[2]

A fast and efficient simulation of modern materials, like (nearly) incompressible nonlinear elastic materials, underlying a large deformation has become more and more interesting in recent years. To avoid the occurrence of the infinite bulk modulus in the corresponding deformation problem, it is a common approach to introduce a new variable, the hydrostatic pressure, and to work with a mixed formulation.

In this talk we will present a formulation to numerically simulate the deformation $\boldsymbol{U}$ and the hydrostatic pressure P on a three-dimensional region Ω by using a mixed, adaptive finite element method. Starting from the equilibrium of forces we will derive the weak form of the deformation problem, which is nonlinear and can be solved with a Newton's method and incremental load steps. In every iteration step this will lead to a saddle point problem. By using Taylor Hood finite elements we will obtain a discrete, indefinite problem, which can be handled with the Bramble Pasciak conjugate gradient method. Finally we want to discuss some modifications to improve the simulation and give some numerical results.

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ALE-FEM for two-phase flows with insoluble surfactants

Andreas Hahn[1] Lutz Tobiska[2]

We present a finite element method for the flow of two immiscible incompressible fluids in two and three dimensions. Thereby the presence of surface active agents (surfactants) on the interface is allowed, which alter the surface tension.

The model consists of the incompressible Navier-Stokes equations for velocity and pressure and a convection-diffusion equation on the interface for the distribution of the surfactant. A moving grid technique is applied to track the interface, on that account a Arbitrary-Lagrangian-Eulerian (ALE) formulation of the Navier-Stokes equation is used. The surface tension force is incorporated directly by making use of the Laplace-Beltrami operator technique. Furthermore, we use a finite element method for the convection-diffusion equation on the moving hyper surface. isoparametric finite elements are used. In order to get a high accurate method the interface, velocity and pressure are approximated by isoparametric finite elements.

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A preconditioned GMRES solver with algebraic multigrid accelerations for the fluid-structure interaction problems

Huidong Yang[1]

In this paper, we propose a preconditioned GMRES method for solving a Schur complement equation of the coupled fluid-structure interaction system, with respect to the displacement unknowns only on the interface. The preconditioning for the interface equation requires approximate solutions of the structure and the fluid sub-problems, with respectively prescribed Robin boundary conditions on the interface for each sub-problem. The solutions of both sub-problems are approximated by very few W-cycles of special algebraic multigrid methods applied to a symmetric and positive definite system and a saddle point system from the discretized structure and the fluid sub-problems, respectively.Both sub-problems are discretized by the finite element method on hybrid meshes. The application of these W-cycles enhances the performance of solving the interface equation.

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A priori error estimates for an elliptic control problem with non-differentiable cost functional

Roland Herzog[1] Eduardo Casas[2] Gerd Wachsmuth[3]

We consider an optimal control problem subject to a semilinear elliptic equation. The objective is non-differentiable due to the presence of the L1-norm of the control. We provide first- and second-order optimality conditions as well as a priori error estimates for various discretizations of the problem.

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Optimal Dirichlet boundary control for the Navier–Stokes equations

Lorenz John[1] Olaf Steinbach[2]

We consider an optimal Dirichlet boundary control problem for the Navier–Stokes equations. The control is considered in the energy space, where the related norm is realized by the so called Steklov–Poincaré operator. Further we introduce a stabilized finite element method for the optimal control problem, where we especially focus on lowest order elements. Finally, we present some numerical results.

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Neumann boundary control of semilinear elliptic equations: finite element discretization and error estimates

Johannes Pfefferer[1] Klaus Krumbiegel[2]

This talk is concerned with the finite element discretization and error analysis of Neumann boundary control problems governed by semilinear elliptic partial differential equations in polygonal domains where the control has to fulfil pointwise inequality constraints. In order to solve this problem the state and the adjoint state are discretized by linear finite elements whereas the control is discretized by piecewise constant ansatz functions. In a postprocessing step approximations of the continuous optimal control are constructed which possess superconvergence properties. The negative influence of corner singularities on the approximation rates is compensated by mesh grading techniques. Imposing second order sufficient optimality conditions for the continuous optimal control, error estimates for the constructed control related to the continuous one is proven. Finally, the quality of the approximations is demonstrated by a numerical example.

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Maximum-norm a posteriori error estimators for parabolic problems

Torsten Linß[1] Natalia Kopteva[2]

For classical and singularly perturbed semilinear parabolic equations, we present maximum norm a posteriori error estimates that, in the singularly perturbed regime, hold uniformly in the small perturbation parameter. The parabolic equations are discretised in time using the backward Euler method, the Crank-Nicolson method and the discontinuous Galerkin dG(1) method. Both semidiscrete (no spatial discretation) and fully discrete cases will be considered. The analysis invokes elliptic reconstuructions and elliptic a posteriori error estimates.

Acknowledgement. This work was supported by Science Foundation Ireland, Grant 08/RFP/MTH1536 (PI N. Kopteva).

References:

[1] N. Kopteva and T. Linß, *A posteriori error estimation for parabolic problems using elliptic reconstructions. I: Backward-Euler and Crank-Nicolson methods* (submitted for publication) http://www.staff.ul.ie/natalia/pdf/Kopt_Linss2011_1.pdf

[2] N. Kopteva and T. Linß, *A posteriori error estimation for parabolic problems using elliptic reconstructions. II: The discontinuous Galerkin method* (in preparation)

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Optimal control of quasistatic plasticity

Gerd Wachsmuth[1]

The optimization of elastoplastic systems is of significant importance for industrial deformation processes, e.g., for the control of the springback of deep-drawn metal sheets. We consider an optimal control problem for the stress-based (dual) formulation of quasistatic elastoplasticity. The control-to-state map can be interpreted as an evolutionary variational inequality. Hence, it is non-differentiable and the analysis has to rely on time-discretization. For the constitutive equations of the elastoplastic material, we use the linear kinematic hardening model and the von Mises yield condition.

Necessary optimality conditions of weak-stationary type are obtained by time-discretization and regularization. Some numerical results concerning the control of the springback are shown.

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Stabilised finite element discretisations applied to an operator splitting method of population balance equations

Naveed Ahmed[1] Gunar Matthies[2] Lutz Tobiska[3]

In population balance equations, the distribution of the entities depends not only on space and time but also on their own properties referred to as internal coordinates. This results into a transient higher dimensional problem. First, an operator splitting method is applied to transform the original problem into two subproblems: a transient transport problem with pure advection and a time-dependent convection-diffusion problem. Then, both subproblems are discretised in time by a backward Euler time stepping scheme. A discontinuous Galerkin methods is applied for discretising the transport problem in the internal coordinate. For the convection-diffusion problem, stabilised finite element methods are used. In particular, the Streamline-Upwind Petrov-Galerkin (SUPG) method and the local projection stabilisation (LPS) method are investigated. We present error estimates for the two-step method and discuss the assumptions on the dependence of the stabilisation parameters on the time step length for both stabilisation methods. We also compare numerically the SUPG and the LPS method.

References:

[1] N. Ahmed: Stabilized finite element methods applied to transient convection-diffusion-reaction and population balance equations, PhD thesis, Otto-von-Guericke-Universität Magdeburg, 2011.
[2] N. Ahmed, G. Matthies, L. Tobiska: Finite element methods of an operator splitting method applied to population balance equations. Preprint 05/2011, Fakultät für Mathematik, Otto-von-Guericke-Universität Magdeburg, 2011.

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A non-standard finite element method for convection-diffusion-reaction problems on polyhedral meshes

Clemens Hofreither[1] Ulrich Langer[2] Clemens Pechstein[3]

We present a new non-standard finite element method based on element-local boundary integral operators that permits polyhedral element shapes as well as meshes with hanging nodes. The method employs elementwise PDE-harmonic trial functions and can thus be interpreted as a local Trefftz method.

The construction principle requires the explicit knowledge of a fundamental solution of the partial differential operator, but only locally, i.e., in every polyhedral element. Fundamental solutions are known for general elliptic PDEs of diffusion-convection-reaction type with constant coefficients. This allows us to solve such PDEs with elementwise constant coefficients.

In this talk, we apply the method to convection-diffusion problems, in particular convection-dominated problems. We show that the PDE-harmonic trial functions lead to improved stability over standard piecewise linear ones and discuss possible further stabilization approaches.

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Convergence and stability in balanced norms of finite element methods on Shishkin meshes for reaction-diffusion problems

Martin Schopf[1] Hans-G. Roos[2]

Error estimates of finite element methods for reaction-diffusion problems are often realized in the related energy norm. In the singularly perturbed case, however, this norm is not adequate. A different scaling of the H^1 seminorm leads to a balanced norm which reflects the layer behavior correctly. We prove an error estimate in a balanced norm and investigate also stability questions.

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The finite cell method: a fictitious domain approach applying high-order finite elements

Alexander Düster[1] Stefan Kollmannsberger[2] Dominik Schillinger[3] Ernst Rank[4]

The Finite Element Method (FEM) has become the most frequently applied numerical method in Computational Mechanics. Although the FEM is in a mature state there are still problems where its application is difficult. These problems arise, for example, when considering heterogeneous materials or more generally when discretizing structures which have a very complex geometry. In such cases mesh generation can become very involved. To overcome these problems we propose to apply the Finite Cell Method (FCM) which can be considered as a combination of a fictitious domain method with high-order finite elements. The main idea is to embed the physical domain into an extended domain which can be easily discretized with structured hexahedral meshes. In this way, mesh generation is dramatically simplified and the burden is shifted towards the integration of the stiffness matrices. However, the integration can be performed adaptively in a fully automatic way. During the integration the geometry and varying material properties are taken into account while the discretization error is controlled by adjusting the polynomial degree of the hexahedrals. The proposed method will be illustrated by considering the discretization of problems of solid mechanics in one, two and three dimensions. We present applications of the FCM to the computation of thin-walled structures, the numerical homogenization of heterogeneous materials as well as problems of topology optimization.

References:

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[2] A. Düster, J. Parvizian, Z. Yang, E. Rank. The finite cell method for three-dimensional problems of solid mechanics. *Comput. Methods Appl. Mech. Engrg.*, 197:3768–3782, 2008.
[3] E. Rank, S. Kollmannsberger, Ch. Sorger, A. Düster. Shell Finite Cell Method: A High Order Fictitious Domain Approach for Thin-Walled Structures. *Comput. Methods Appl. Mech. Engrg.*, doi:10.1016/j.cma.2011.06.005, 2011.
[4] J. Parvizian, A. Düster, E. Rank. Topology optimization using the finite cell method. *Optim. Eng.*, DOI 10.1007/s11081-011-9159-x, 2011.

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Schwarz type solvers for hp-FEM discretizations of mixed problems

Sven Beuchler[1] Martin Purrucker[2]

The Stokes problem and linear elasticity problems can be viewed as a mixed variational formulation. These formulations are discretized by means of the hp-version of the finite element method. The system of linear algebraic equations is solved by the preconditioned Bramble-Pasciak conjugate gradient method. The development of an efficient preconditioner requires three ingredients, a preconditioner related to the components of the velocity modes, a preconditioner for the Schur complement related to the components of the pressure modes and the discretization by a stable finite element pair which satisfies the discrete inf-sup condition. The last condition is also important in order to obtain a stable discretization scheme. The preconditioner for the velocity modes is adapted from fast hp-FEM preconditioners for elliptic problems. Moreover, we will prove that the preconditioner for the Schur complement can be chosen as a diagonal matrix if the pressure is discretized by discontiuous finite elements. We will prove that the system of linear algebraic equations can be solved in almost optimal complexity if the $Q_k - P_{k-1,disc}$ element is used. This yields to quasioptimal hp-FEM solvers for the Stokes problems and linear elasticity problems. The latter are robust with respect to the contraction ratio ν. The efficiency of the presented solver is shown in several numerical examples.

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A fully automatic *hp*-adaptive refinement strategy for Maxwell's equations

Markus Bürg[1]

The performance of the classical finite element method can be improved by mesh refinement (h-refinement) or the use of higher order ansatz spaces (p-refinement). A combination of both (hp-refinement) can lead to exponential convergence of the computed solution.
Nowadays adaptive refinement of the computational domain is a widely used feature to obtain an accurate numerical solution of the partial differential equation with as less computational work as possible. Therefore one needs to decide, where the approximation error of the numerical solution is relatively large and, thus, refinement should take place. Since the analytic solution is ussually not known, one has to estimate the approximation error in terms of the numerical solution to be able to decide, which areas of the computational domain have to refined further.
In recent years a broad interest in the numerical solution of Maxwell's equations has come up, because this system of equations appears in a lot of nano-scaled processes due to the presence of an electromagnetic field. Solving these equations numerically usually requires a lot of computational work and a fully hp-adaptive refinement strategy can reduce the amout of work, which is related to solving the stationary problems, significantly. We present an hp-efficient residual-based a posteriori error estimator for Maxwell's equations in the electric field formulation, which gives a reliable and robust estimation of the true energy error. Then an hp-refinement strategy is introduced, which is based on the solution of local boundary value problems.
These are the major ingredients for a fully automatic hp-adaptive refinement algorithm, which creates a sequence of problem-adapted finite element approximation spaces allowing the convergence of the finite element solution towards the analytic solution with an exponential rate of convergence.

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Generic construction of high order finite-element spaces

Andreas Dedner[1] Martin Nolte[2]

In this talk we will discuss the implementation of a large range of high order finite-element shape functions. The construction principle closely follows the well known abstract definition as presented for example in the textbook on finite-elements by Ciarlet. Most notably the implementation often only requires the description of the nodal variables, the construction of the actual shape functions is carried out automatically.

In many cases the nodal variables can be generically implemented for a wide range of reference elements and polynomial degrees using the recursive construction of a set of reference elements as found in the DUNE software framework (www.dune-project.org). We have used this approach to implement different Lagrange type finite-element spaces, different DG spaces, and vector valued Raviart-Thomas type finite-elements. The automatic generation of the basis functions makes it easy to use different sets of nodal variables to describe the finite-element space. We will demonstrate the effectiveness of our approach by testing the conditioning and approximation quality of different versions of high order Raviart-Thomas type elements.

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Frequency selection in heaving airfoil wakes using a high-order numerical method

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The unsteady flow over streamlined bodies is one of the most important problems in fluid dynamics. The interest for these flows is motivated by their highly non-linear and unsteady nature, which make them almost impossible to solve analytically. With the increase in computing power the use and development of high-order computational methods has become an attractive alternative for the numerical modelling of unsteady flows over streamlined configurations. In this work we conduct numerical simulations of steady and unsteady flows over motionless and heaving airfoils using a high-order Spectral Element method for the first time. Our simulations confirm the suitability of this method to model and characterize in very good spatial and temporal detail the flow structures and wake transitions. The results are validated against previously published experimental and computational studies. Heaving airfoils shed vortices as they oscillate, and these wakes are classified into drag, neutral and thrust-producing wakes, depending on the nature of the force produced by the airfoil. In this work drag, neutral and thrust wakes are successfully simulated, and also the transitions from one wake to another. Two new modes are observed in this investigation and added to the wake classification. We question the assumption that the Strouhal number is the main and only parameter to characterize the wake configurations, and thus the nature of the forces produced for oscillating airfoils. Our findings show that, in order to characterize such flows one needs to consider the amplitude and frequency of oscillations as independent parameters, and that the Strouhal number alone is not sufficient to characterize oscillating airfoil wakes. Finally, we explore the frequency regimes for oscillating airfoils. These regimes depend on the forcing frequency and the forcing amplitude and on the relation between the forcing frequency and the natural frequency of the airfoil. Three frequency regimes are defined in the literature: the natural regime, the harmonic regime and the lock-in regime. These different frequency regimes are successfully simulated in this investigation. They are related to the shedding process through which the wake undergoes a transition from a Karman street to a reversed Karman street. The transition between the different frequency regimes is simulated at both constant frequency and constant amplitude. We found that the frequency regimes are strongly related to the wake type exhibited. Wake-types with multiple-vortices-per-half-cycle of oscillation are found in harmonic regimes and wake-types with one vortex-pair shed per cycle are in the region where one distinct frequency is in control (lock-in and natural regimes). Wakes where the leading-edge vortices contribute to the shedding process are also simulated.

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Comparison of hp-adaptive finite element strategies

William Mitchell[1] Marjorie A. McClain[2]

Adaptive finite element methods have been studied for nearly 30 years now. Most of the work has focused on h-adaptive methods where the mesh size, h, is adapted locally by means of a local error estimator with the goal of placing the smallest elements in the areas where they will do the most good. h-adaptive methods for elliptic partial differential equations are quite well understood now, and widely used in practice. Recently, the research community has begun to focus more attention on hp-adaptive methods where in addition to h-adaptivity one locally adapts the degree of the polynomials, p. One attraction of these methods is that they can achieve exponential rates of convergence [1]. But the design of an optimal strategy to determine when to use p-refinement, when to use h-refinement, and what p's to use in h-refined elements is an open area of research. Many such hp-adaptive strategies have been proposed over the past two decades [2]. In this talk, we will briefly describe 13 hp-adaptive strategies and present the results of a numerical experiment to determine which strategies are most effective in terms of error vs. degrees of freedom in different situations.

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Higher order variational time discretizations for nonlinear systems of ordinary differential equations

Gunar Matthies[1] Friedhelm Schieweck[2]

We discuss different time discretizations of variational type applied to a nonlinear system of ordinary differential equations which is generated by a semi-discretization in space of a given nonlinear parabolic partial differential equation like, for instance, the non-stationary Burgers equation.

Among these methods we compare the known continuous Galerkin-Petrov and the discontinuous Galerkin method with time polynomial ansatz functions of order k (cGP(k)- and dG(k)-method) with respect to accuracy, stability and computational costs. Moreover, we propose two new extended methods (cGP-C1($k+1$)- and dG-C0($k+1$)-method) which have on the one hand a one higher degree of ansatz functions and accuracy, the same stability properties and on the other hand the same computational costs as the original methods.

We present optimal error estimates and the close relationship between the original and extended methods which prove as a byproduct the super-convergence of the original methods cGP(2) and dG(1) in the endpoints of the discrete time intervals. Finally, we present first numerical results for the non-stationary Burgers equation in one space dimension.

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Efficient numerical realization of discontinuous Galerkin methods for temporal discretization of parabolic equations

Andreas Springer[1] Thomas Richter[2] Boris Vexler[3]

Time discretization of nonlinear parabolic equations using discontinuous Galerkin methods of order r results in a coupled system of $r+1$ elliptic problems for every time step which have to be solved by Newton's method. Since assembling and solving this system tends to be challenging, both in terms of memory consumption and in terms of adopting existing finite element code for this purpose, it is desirable to decouple the Newton update equation. A direct decoupling would lead to elliptic sub-problems with complex coefficients. We circumvent this problem by replacing the Newton update equation with a suitable approximation resulting from block Gaussian elimination and subsequent approximation of the last block of the obtained block upper triangular matrix. The resulting solution schemes do not have superlinear convergence, but rapid linear convergence is proved. All steps involved in the proposed scheme have the same structure as implicit Euler steps for the considered problem, in particular no large block systems have to be assembled and no complex arithmetic is required. Numerical tests show that the required computing time compares favourably to assembling and solving the complete coupled system by Newton's method and memory consumption is reduced.

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A posteriori control of modeling and discretization errors in finite elastoplasticity

Andre Große-Wöhrmann[1] Heribert Blum[2]

The concept of adaptive error control for finite element Galerkin discretizations has more recently been extended from the pure treatment of the discretization errors [1], [4] also to the control of modeling errors [6]. These techniques can be employed for a rigorous justification of the local choice of the model out of a given hierarchy with increasing complexity. In the present paper the concept is exemplified by a hierarchy of elastoplasticity models [2], [3], [7]. Significant reduction of the computational complexity can be achieved by a proper choice of the model in different subdomains, automatically chosen by the error estimators. The method is applied to finite element simulations [5] of metal forming.

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Condition number estimates for the finite element method on adaptive meshes

Lennard Kamenski[1] Weizhang Huang[2] Hongguo Xu[3]

It has been demonstrated that significant improvements in accuracy can be gained when an appropriately chosen anisotropic mesh is used in the numerical solution of PDEs exhibiting anisotropic features. However, an anisotropic mesh could contain elements of large aspect ratio and very small volume and, consequently, there exists a concern in the numerical analysis community that an adaptive (anisotropic) mesh can lead to linear algebraic systems which are ill-conditioned and difficult to solve and this may outweigh the accuracy improvements gained if using an anisotropic mesh.

The objective of this work is to study the condition number of linear systems resulting from linear FEM discretization on adaptive meshes. We use the technique introduced by I. Fried and compute bounds on the condition number of the stiffness matrix for arbitrary meshes with the help of an appropriately chosen fictitious density distribution. The estimates involve eigenvalues of the mass matrix and the principal eigenvalue of the structure.

A surprising result is that the condition number on adaptive meshes is much better than generally assumed, especially in two dimensions.

We further develop a simple diagonal scaling which employs available mesh information and significantly improves the condition number of the resulting linear system.

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An adaptive DG finite element method in the space-time domain

Martin Neumüller[1] Olaf Steinbach[2]

For evolution equations we present a flexible space-time method based on Discontinuous Galerkin finite elements. Space-time methods have advantages when we have to deal with moving domains and if we want to do local refinement in the space-time domain. For this we use a residual based error estimator. This method will be applied to the heat equation and to the Navier Stokes equations. Numerical examples and some applications will be given.

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Simulation of fibre reinforced polymers with adaptive FEM

Michael Weise[1] Arnd Meyer[2]

Lightweight construction is an important approach to increase energy efficiency. The usage of composite materials like fibre reinforced polymers (FRP) plays an important role in lightweight construction. FRP can be characterised by transversely isotropic material behaviour, a special case of anisotropy. Utilizing a homogenisation of the material as a continuum no modelling of the single fibres is needed.

A precise simulation of FRP components and safe failure criteria are the necessary basis for efficient design of lightweight structures. A fast as well as highly accurate computation can be achieved using the adaptive finite element method with its solution-dependent automatic mesh refinement.

In our talk we present the necessary efforts to include such material behaviour into an existing adaptive FEM code. We will show some results of our simulation of FRP with the adaptive finite element method.

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Mixed finite element methods for radiative transport

Herbert Egger[1] Matthias Schlottbom[2]

The radiative transfer equation (RTE) is a basic model for transport, absorption and scattering of particles, e.g., photons, in dense media. We consider the numerical solution of the RTE via Galerkin discretizations of a mixed variational principle. We prove inf-sup stability conditions on the continuous and discrete level. Two particular discretizations of PN and SN type are presented, and limitations of these approaches, arising from the discrete inf-sup conditions, are highlighted.

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Axisymmetric problems in continuum mechanics with transversely isotropic materials

Hansjörg Schmidt[1] Arnd Meyer[2]

We investigate problems consisting of a axisymmetric body under axisymmetric load with radial, longitudinal and rotational degrees of freedom. The fundamental equations are deduced in a short and elegant way. The decoupling of the rotational degree of freedom from both other is derived for isotropic materials and two quite limiting cases of transversely isotropic material. In a straightforward way we apply error estimators, adaptive refinement and hierarchical preconditioning to achieve a fast solver for this important class of problems.

Furthermore we investigate a hydrogen tank under internal pressure. The tank consists of a steel liner and several layers of carbon rovings with different fibre directions. Due to the process of winding the rovings around the liner, the fibres are geodesics, with respect to some intermediate surface. As a consequence of Clairaut's relation $R(\zeta)\cos\varphi(\zeta) = \text{const}$ the maximum altitude ζ of a geodesic is bounded if the radius R tends to zero. This influences the geometry of the tank and some post processing is shown for the specified problem.

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Basic modelling for large deformation on plates

Jens Rückert[1] Arnd Meyer[2]

We investigate large deformations of plates with nonlinear elastic material. Therefore we consider a model using the Kirchhoff assumption locally, avoiding any further simplifications. This way of modelling leads to a two-dimensional strain tensor, which depends essentially on the two fundamental forms of the differential geometry of the deformed midsurface. The desperate ambitious Newton linearization of the arising equation is analytically very expensive. So we examine replacements by some modified Newton linearizations.

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A plate equation for elasticity with large deformations

Andreas Günnel[1]

We present a model for large deformations of plates, involving non-linear elastic material behaviour. Unlike most models using the Kirchhoff hypothesis, changes in thickness and shear are feasible, thus the model does not limit to very thin plates . However, a limit process of vanishing thickness yields parallels to the Kirchhoff hypothesis. The model is applied to contact problems involving a penalty approach which can be interpreted to approximate the deformation energy of the obstacle.

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FEM-simulation of crack propagation in linear piezoelectric material

Peter Steinhorst[1]

By far most of the piezoelectric devices nowadays consist of ceramics, which are known as brittle material. Therefore, cracks are a well-known phenomena in piezoelectrics. Of special interest is the question of crack propagation. Numerical Methods which are able to predict if a crack propagates or not, and if propagate which crack path can be expected, are of practical interest in order to design potential endangered devices etc.

Although the cracking process in piezoelectric materials is not completely understood up to now, especially the influence of the electric field to possible cracking criterions in general (see, e.g. [3]), some advances have been made on this topic in the last decades.

In a cooperation project there has been developed a simulation tool, which allows to test different cracking criterions in numerical examples with linear piezoelectric material[1]. At each cracking step an adaptive FEM-solution matching to the actual crack configuration is computed, and in postprocessing a calculation of certain parameters is performed (which are used in cracking criterions to indicate or quantify crack propagation). After a crack increment, the procedure repeats. The basics for the FEM-simulation are explained in some details in [4]. Results of such numerical tests can be compared with results of real experiments under defined loads.

Of course, the assumptions of the model used in the simulation method are a restriction to real-world behaviour of piezoelectric ceramics. Several enhancements might help to lead the simulation closer to reality, as example the assumption of more general than impermeable boundary conditions on the crack faces or the possibility of contact between crack faces have been examined[2] (but complicate the simulation significantly).

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Multiphase field modeling chemical vapor infiltration

Yaochan Zhu[1] Eckart Schnack[2]

The chemical vapor infiltration (CVI) is an industrially widely used process to produce the SiC matrix composites reinforced by SiC fibers in ceramic engineer. To reproduce the complicated microstructure evolution during this process, a multiphase field model is formulated. The model consists of a set of nonlinear partial differential equations by coupling Ginzburg-Landau type phase field equations with mass balance diffusion equation and Navier-Stokes fluid field equation. Mathematically, the co-deposition of SiC, Si and C from methyltrichlorosilane(MTS) during CVI process is a typical free boundary problem. When this problem is handled by multiphase field method, it's greatly facilitated because of the implicitly boundary-tracking technique by phase field order parameters. This typical multiphysic field problem is solved by continuous finite element method with software COMSOL. The numerical result shows good agreement with experimental investigation.

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Parallel acceleration: achievements and pitfalls

Gundolf Haase[1]

Medium size computer clusters with several hundred CPU-cores are already available to small and medium size companies. The bunch of cores doesn't perform always as well as expected for several reasons and these miracles continue when GPUs are used as additional performance booster.

Based on several examples we will show great parallel (strong) speedup on multiple cores and on clusters of compute nodes as well as (a few) discouraging results that might have to be solved by means of mathematics. Especially the GPU parallelization combines all sorts of pitfalls from shared and distributed memory computing.

We will present several student projects from physics and engineering and how they advanced during the winter term. The examples range from non-Newtonian fluids for shared memory systems, or PDE solvers on GPU to a (random walk) worm algorithm for Lattice-Boltzmann on GPU. The performance gain on GPUs ranged from dissatisfying (factor 3) for a standard package to splendid (factor 300) in case of quantum chromodynamic problem. The typical acceleration by a GPU is a factor of 10 for memory intense algorithms and a factor 50 and more for compute intense algorithms.

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