On approximation of higher order for variational inequalities of mixed type

Joachim Gwinner[1]

In this talk we are concerned with the finite element method in its p-version to treat a scalar variational inequality of the mixed type that models unilateral contact and Coulomb friction or other nonsmooth material behaviour in continuum mechanics.

This leads to a nonconforming discretization scheme. In contrast to previous work we employ Gauss-Lobatto quadrature for the approximation of the unilateral constraint and also for the friction-type functional. We take the resulting quadrature error into account of the error analysis.

At first without any regularity assumptions, we prove convergence of the FEM Galerkin solution in the energy norm. To this end we investigate Mosco-Stummel-Glowinski convergence [3] for both convex sets and convex functionals. The key of our norm convergence result for the $p-$FEM is the used Gauss-Lobatto integration rule with its high exactness order and its positive weights together with duality arguments in the sense of convex analysis.

Secondly by a novel Céa-Falk lemma we split the total discretization error into two different parts: the distance of the continuous solution to the convex set of approximations in the trial space and the consistency error caused by the nonconforming approximation. Here we use the well-known approximation theory of spectral methods [1], the cutting technique of Falk [2], and interpolation arguments. Thus we arrive under mild regularity assumptions at an a priori error estimate which is suboptimal because of the treatment of the consistency error in the nonconforming approximation scheme and because of the regularity threshold in unilateral problems.

References:

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[1] Universität der Bundeswehr München, Aerospace Engineering,
`Joachim.Gwinner@unibw-muenchen.de`