Reduced basis methods for parametrized variational inequalities

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Reduced Basis Methods are increasingly popular methods for simulation-based model reduction of parametrized problems. Different types of partial differential equations have been treated, in particular linear and nonlinear, coercive and non-coercive stationary problems, as well as parabolic and hyperbolic evolution problems. Systems can also be dealt with, such as Stokes, Navier-Stokes and Boussinesq equations, etc. see [1] and references therein. Key ingredients are snapshot-based reduced basis space construction with (POD)-greedy techniques [2], rigorous a-posteriori error control by residual analysis, and rapid online simulations by decoupling into an offline and online simulation phase. In the current presentation, we address a new class of problems, which are given by combining partial differential equations with inequality constraints [3]. These can be obtained from constrained variational problems, such as occurring in contact problems. We give a formulation of a reduced basis method for parametrized variational inequalities. We explain special considerations in reduced basis construction for primal and dual variables, and comment on the computational procedure for the reduced problem. The reduced problem allows a full offline/online decomposition, which enables rapid online simulations independent of the dimension of the full problem. We give several analytical results for the reduced solutions such as well-posedness, boundedness by the data functions and Lipschitz-continuity with respect to the parameters. A-posteriori error estimation is more difficult than in the pure equality context, still it is possible to derive rigorous error bounds. The a-posteriori error estimators can for instance be used in the offline-phase for reduced basis generation with greedy procedures. Numerical examples demonstrate different aspects of approximation quality and computational time of the reduced scheme and the quality of the error estimators.

References:

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