Efficient numerical realization of discontinuous Galerkin methods for temporal discretization of parabolic equations

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Time discretization of nonlinear parabolic equations using discontinuous Galerkin methods of order r results in a coupled system of $r+1$ elliptic problems for every time step which have to be solved by Newton's method. Since assembling and solving this system tends to be challenging, both in terms of memory consumption and in terms of adopting existing finite element code for this purpose, it is desirable to decouple the Newton update equation. A direct decoupling would lead to elliptic sub-problems with complex coefficients. We circumvent this problem by replacing the Newton update equation with a suitable approximation resulting from block Gaussian elimination and subsequent approximation of the last block of the obtained block upper triangular matrix. The resulting solution schemes do not have superlinear convergence, but rapid linear convergence is proved. All steps involved in the proposed scheme have the same structure as implicit Euler steps for the considered problem, in particular no large block systems have to be assembled and no complex arithmetic is required. Numerical tests show that the required computing time compares favourably to assembling and solving the complete coupled system by Newton's method and memory consumption is reduced.

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