A posteriori analysis for phase field simulations in the sharp interface limit

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Phase field equations are widely used to approximate the evolution of free moving interfaces. The diffuse interface width ϵ is related to a singular perturbation leading to nonlinear parabolic equations.

Efficient finite element simulations of phase field problems require a fine grid resolution only close to the moving fronts. Therefore, adaptive mesh refinement and coarsening is highly desirable. Unfortunately, conventional a posteriori error estimators depend exponentially on the inverse ϵ^{-1} of the interface parameter and become useless in the sharp interface limit $\epsilon \to 0$.

We derive robust and fully computable error estimators that depend on ϵ^{-1} only in a low order polynomial. They are based on the numerical evaluation of the principal eigenvalue of the linearized operator. Numerical results show that robust error control is possible, even if the phase field solution undergoes topological changes, which causes a blow–up of the principal eigenvalue when $\epsilon \to 0$.

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