On optimal complexity gradient type solvers for elliptic eigenvalue problems

Klaus Neymeyr[1]

The topic of this talk is the numerical solution of eigenvalue problems for self-adjoint elliptic partial differential operators. Various gradient iterations can be applied to the finite element discretization in order to compute some of the smallest eigenvalues and its corresponding eigenfunctions.

First we present new convergence estimates for a class of (non-preconditioned) gradient iterations [1]. The analysis is based on a theorem which shows that poorest convergence within Krylov subspaces is attained in invariant subspaces of small dimensions. Then sharp convergence estimates for the Rayleigh-Ritz approximations are proved by means of a *mini-dimensional analysis*. However, these simple gradient iterations show a mesh-dependent convergence rate.

In a second step *preconditioning* is introduced by changing the geometry underlying these gradient iterations. With suitable multigrid preconditioners optimal complexity solvers are attainable which allow the numerical approximation of a fixed number of the smallest eigenvalues and the corresponding eigenfunctions with costs that increase linearly in the number of unknowns. These solvers can be derived systematically by a comparison of the boundary value and of the eigenvalue problem for the same partial differential operator. Until now only a small number of convergence rate estimates are known for these iterations. Only for the simplest solver - the fixed step size preconditioned gradient iteration - sharp convergence estimates are known. A new *gradient-flow-analysis* [2] is presented which allows to derive sharp convergence rate estimates in a significantly simpler and shorter way compared to the original proof.

References:

[1] K. N. and M. Zhou: *Convergence analysis of gradient iterations for the symmetric eigenvalue problem*, Technical report 2010.
[2] A. Knyazev and K. N.: *Gradient flow approach to geometric convergence analysis of preconditioned eigensolvers*, SIAM J. Matrix Analysis 31 (2009), 621–628.

[1] Institut für Mathematik, Universität Rostock,
klaus.neymeyr@uni-rostock.de