Convergence analysis of iterative methods for algebraic nonlinear eigenvalue problems

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The convergence analysis of iterative methods for algebraic nonlinear eigenvalue problems is in the most cases restricted to simple eigenvalues. In this talk the convergence order for multiple eigenvalues for several methods as the inverse iteration and Kummer's method is analyzed. We use the Smith form of matrix functions to characterize the multiplicity of the eigenvalues. The key tool of our analysis is a special representation of the resolvent close to an eigenvalue as Laurent series which was introduced by Keldysh for polynomial eigenvalue problems and extended by Gohberg and Sigal for meromorphic ones. We show that the convergence rate of the methods depends on which order the eigenvalues have as pole of the resolvent. For the inverse iteration and Kummer's method local quadratic convergence is obtained if the eigenvalue is semi-simple. For defective eigenvalues both methods have only a local linear convergence order.

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