Weighted Poincare inequalities and robust domain decomposition solvers

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Robust solvers for problems with high-contrast coefficients are currently an important and active area of research. In this talk we present weighted Poincaré inequalities of the form

$$\inf_{c\in\mathbb{R}} \int_\Omega \alpha(x)\,|u(x)-c|^2\,dx \;\leq\; C_P\,\mathrm{diam}(\Omega)^2 \int_\Omega \alpha(x)\,|\nabla u(x)|^2\,dx$$

for functions u in $H^1(\Omega)$ or in a finite element space. For a certain class of piecewise constant and positive weight functions $\alpha(x)$, we can get the constant C_P independent of the values of α, i. e. of high contrast in α.

As a simple example consider the case where α takes two different values on two connected subregions $\Omega^{(k)}$ of Ω. For this situation we can even give estimates on how C_P depends on the subregions $\Omega^{(k)}$. Generalizations to the multi-subregion case are also possible.

Finally, we give some applications in domain decomposition methods, in particular for FETI (finite element tearing and interconnecting) type methods. With our inequalities we can show condition number estimates that are robust for certain high contrast coefficients, including cases where the subdomain partitioning does not resolve coefficient jumps.

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