Numerical modeling eutectic solidification by multiphase field technique

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Here the multiphase field model[1,2] is studied which is extensively applied in simulation of microstructure evolution during phase transformations such as solid-solid transition, solidification, grain growth. Depending on problem itself, the model is openly coupled by a set of partial different equations which is of property of conserved nonlinear parabolic equations (Cahn-Hilliard type[3]) for temperature or composite and non-conversed hyperbolic equations (Cahn-Allen type[4]) for phase field order parameters. The basic physic meaning behind multiphase phase field model is that supposing the system free energy tends toward minimization by the response of phase field order parameters to the driving force. As a diffusive interface model, the movement of interfaces/boundaries which involved in complex microstructure evolution during transformation/grain growth is implicitly tracked by multi-phase field parameters, which changes smoothly across a spatial diffuse interface/boundary from one constant to another within finite width. This character greatly facilitates dealing with free boundary problem (e. g. eutectic solidification), which is mathematically described as a sharp interface model requiring explicit tracking interfaces/boundaries with great difficulty by numerical method.

The finite difference method is of advantage over other numerical method such as finite element method, spectral method that it is conceptually intuitive and easy to implement, and consequently, it is most frequently employed for numerical work with multiphase field model[5]. Here this method is applied to discretize the set of partial differential equations of the multiphase model for eutectic solidification and it is analyzed including its consistence, stability and convergence. The detailed discretization error analysis is made. One numerical example of multiphase phase field modeling is supplied to show this process.

References:

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