Preconditioned inverse iteration for $\mathcal{H}$-matrices

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We will present a method of almost linear complexity to approximate some (inner) eigenvalues of symmetric differential operators. Using $\mathcal{H}$-arithmetic the discretisation of the operator leads to a large hierarchical ($\mathcal{H}$-) matrix M. We assume that M is symmetric, positive definite. Then we will compute the smallest eigenvalues with Preconditioned Inverse Iteration (PINVIT), which was extensively investigated by Knyazev and Neymeyr.

Hierarchical matrices were introduced by W. Hackbusch in 1998. They are data-sparse and require only $\mathcal{O}(nk \log n)$ storage. There is an approximative inverse, beside other matrix operations, within the set of $\mathcal{H}$-matrices, which can be computed in linear-polylogarithmic complexity. We will use the approximative inverse as preconditioner in the PINVIT.

Further we will combine the PINVIT with the folded spectrum method to compute inner eigenvalues M. We apply PINVIT to the matrix

$$M_\mu = (M - \mu I)^2 .$$

The matrix M_μ is symmetric, positive definite, too.

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