An application of FEM methods to an elliptic optimal control problem with state constraints

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In this talk we examine a discretisation of an elliptic optimal control problem with state constraints, using finite element spaces.
The continuous problem consists of the following task:
We minimise a quadratic functional

$$J(y,u) = \frac{1}{2}\left\|y - y_d\right\|^2_{L^2(\Omega)} + \frac{\nu}{2}\left\|u\right\|^2_{L^2(\Omega)},$$

depending on the control u and the state $y = y(u)$, given as the solution of the Poisson equation with u as the right-hand side and homogeneous Dirichlet boundary data on the smooth domain Ω. y_d is a given function, $\nu > 0$ a fixed parameter. There are further constraints on u given by real numbers a, b and the pointwise inequality $a \leq u(x) \leq b$. Besides, there is the pointwise state constraint $y \geq y_c$ with given smooth function y_c.
We discretise this problem with the aid of piecewise constant functions for the control u and piecewise linear functions for the state y. This leads to an a-priori error estimate for the approximation of the unique global solution u to the above problem in the L^2-norm. As it turns out, we arrive at an order of $\mathcal{O}(h^{3/4})$, which was also tested numerically.

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