On stabilized higher order approximation of time dependent problems

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$$\partial_t u_i + \boldsymbol{b} \cdot \nabla u_i - \nabla \cdot (\boldsymbol{A}_i \nabla u_i) + r_i(\boldsymbol{u}) = f_i \,, \qquad i = 1, \dots, m \,, \tag{1}$$

with $\boldsymbol{u} = (u_1, \dots, u_m)$ are studied in various technical and environmental applications. The reliable approximation of such systems is still a challenging task. The model equations are strongly coupled such that inaccuracies in one unknown directly affect all other unknowns. In large chemical systems with complex interactions numerical artifacts can lead to wrong predictions. Higher order finite element methods reduce the effect of numerical diffusion leading to an artifical mixing of chemical species.

In the convection- and/or reaction-dominated case with solutions having sharp layers, standard finite element methods cannot be applied. Modified finite element approaches are required that are able to prevent unphysical oscillations. Here, we analyze approximating the system (1) in space by higher order finite element methods with *streamline upwind Petrov-Galerkin (SUPG)* and *(an-)isotropic shock capturing stabilization*. The shock capturing terms further reduce spurious localized oscillations in crosswind-direction. Recently, these stabilization techniques were studied for linear finite element methods. However, in combination with higher finite element methods the stabilizations show to be more efficient. The design of the various stabilization parameter is considered carefully.

For a steady nonlinear model problem the error estimate

$$|||u - u_h|||^2 + \sum_{T \in \mathcal{T}_h} \tau_T(u_h) \left\| \boldsymbol{D}_{\rm sc}^{1/2} \nabla u_h \right\|_{L^2(T)}^2 \leq C_{\rm SC} \sum_{T \in \mathcal{T}_h} \frac{h_T^{2(l_T-1)}}{p_T^{2(k_T-1)}} M_T^{\rm opt} \|u\|_{H^{k_T}(T)}^2$$

is shown within an *hp* finite element framework. The efficiency and robustness of the discretization schemes is studied and illustrated by numrous numerical experiments.

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