Hp-time discontinuous Galerkin method for american put option pricing

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High order methods are very efficient to obtain high accuracy with only moderate degrees of freedom. Hence they are well suited for problems where the computational time and the total error are crucial properties. The "fair" price of an American put option can be obtained by solving a parabolical obstacle problem, a modification of the original Black-Scholes PDE.
In this talk we present a *hp*-FE time discontinuous Galerkin method for the parabolical obstacle problem of pricing American put options. The non-penetration condition is resolved using a Lagrange multiplier yielding a mixed formulation. Its Lagrange multiplier space is spanned by basis functions (in space and time variables) which are biorthogonal to the corresponding basis functions for the primal variable. This biorthogonality allows a component-wise decoupling of the weak contact constraints and can therefore be equivalently rewritten in finding the root of a semi-smooth penalized Fischer-Burmeister non-linear complementary function. The arising system of non-linear equations are solved by a globalized semi-smooth Newton algorithm which is proven to converge locally Q-quadratic. Our numerical examples confirm the superiority of this method in terms of error reduction and computational time.

References:

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