Linearization of rational eigenvalue problems arising in band structure computations for photonic chrystals

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This talk is concerned with numerical methods for nonlinear eigenvalue problems of the form $T(\lambda)x = 0$, $x \neq 0$, where the entries of the matrix T are rational functions in λ. This is motivated by electronic band structure calculations for photonic chrystals, where the nonlinear dependence on the eigenvalue parameter arises from a frequency-dependent material model.

The most straightforward approach to solve such eigenvalue problems is to multiply the entries of T by their common denominator and apply a standard linearization to the resulting polynomial eigenvalue problem. However, with an increasing number of poles, the degree of the intermediate polynomial eigenvalue problem quickly becomes large, leading to a severe magnification of the problem size during the linearization process.

In this talk, an alternative approach is proposed based on recent work by Bai and Su. This approach leads to significantly smaller linearizations, especially when the coefficient matrices associated with the rational terms are of low rank. This approach is combined with structure-exploiting Krylov subspace techniques and applied to photonic chrystals composed of Lorentz materials, where a discontinuous Galerkin scheme is used to discretize the underlying PDE eigenvalue problem.

References:

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