Special finite element shape functions based on component mode synthesis

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The finite element solution of

$$\begin{cases} \text{find } u \in H_0^1(\Omega) \ \text{ such that} \\ \int_\Omega \nabla v(x) \cdot (c(x)\nabla u(x))dx = \int_\Omega f(x)v(x), \quad \forall v \in H_0^1(\Omega) \end{cases} \tag{1}$$

has been the subject of much research. Difficulties arise when the coefficient c associated with the second order linear elliptic operator is rough or highly oscillating so that a naive application of the finite element method necessitates a highly refined mesh.

We will present a new finite element discretization for problem (1). Our discretization is based upon the classic idea of component mode synthesis and exploits a decomposition of $H_0^1(\Omega)$ into subspaces orthogonal for the inner product defined by

$$a(u, v) = \int_\Omega \nabla v(x) \cdot (c(x)\nabla u(x))dx. \tag{2}$$

This decomposition is a well-known result, at the heart of the analysis and development of domain decomposition methods for elliptic partial differential equations and modern component mode synthesis methods for the numerical solution of the global eigenvalue problem.

Combining this space decomposition with local eigendecompositions results in our new special finite element method. Numerical experiments illustrate the effectiveness of the proposed shape functions.

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