Discrete maximum principles for linear and higher-order finite elements

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This talk surveys the general theory for the discrete maximum principles (DMP) for the linear second order elliptic problems discretized by the finite element method. We concentrate on the definition of the discrete Green's function (DGF) and its properties. We show that the validity of the DMP is equivalent to the nonnegativity of the DGF. Special emphasis will be given to the case of nonhomogeneous Dirichlet and Robin boundary conditions.

This general theory can be applied to the case of linear finite elements yielding the well-known maximal angle condition for the corresponding mesh. The theory can be equally well applied to higher-order finite elements, but the analysis of the nonnegativity of the DGF is more complicated. However, for triangular elements, we present numerical experiments, where we test the nodal values of the higher-order DGF. The results indicate that the higher-order approximations allow for weaker angle conditions then the linear approximations. In addition, the maximal angle condition for the linear elements seems to reshape into the minimal angles condition for the higher-order case.

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