Adaptive FEM with local Trefftz trial functions for elliptic equations

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We discuss a special finite element method that solves the stationary isotropic heat equation with Dirichlet boundary conditions on arbitrary polygonal and polyhedral meshes. The method uses a space of locally harmonic trial functions to approximate the solution of the boundary value problem. According to this choice, we obtain a variational formulation on the skeleton of the domain. This formulation contains one Steklov-Poincaré-Operator for each element. These operators are constructed by means of boundary integral formulation. Therefore, the proposed finite element method can be used on general polygonal non-conform meshes. Hanging nodes are treated quite naturally. The material properties are assumed to be constant on each element. We also discuss adaptive mesh refinement to handle cases, when the material properties are given as a continuous function.

In a second step we have a look at a posteriori error estimates which can be used for further mesh refinement. Standard methods are based on triangular or quadrilateral meshes. The challenging part is to handle the arbitrary polygonal and polyhedral meshes. Therefore, we make use of functional analytic estimates to overcome these problems.

References:

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