Structure exploiting adjoints

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In this presentation, we consider the numerical solution of PDE constrained optimal control problems of the form

$$
\begin{aligned}
J(u,q) = j(u(T)) + R(q) \rightarrow \min!\\
s.t.\ \partial_t u + \nabla \cdot f(u) \quad &= S(q) \qquad in\ \Omega \\
f(u) \cdot n \quad &= f^b(u) \qquad on\ \Gamma \\
u(0,x) \quad &= u_0(x)
\end{aligned}
$$

with the nonlinear flux $f : \mathbb{R} \rightarrow \mathbb{R}$ and the state $u : [0,T] \times \Omega \subset [0,T] \times \mathbb{R}^d \rightarrow \mathbb{R}$. The normal direction vector is denoted by n. The distributed control is given by $q : [0,T] \rightarrow \mathbb{R}$. Regularization terms are represented by $R(q)$. The boundary of Ω is denoted by Γ. We apply a discontinuous Galerkin method to transform the PDE into an ODE. Furthermore, the integrals in the ODE are replaced by quadrature rules and the time stepping procedure is performed by an explicit Runge-Kutta method. To compute the gradient required for a calculus-based optimization one may use Algorithmic Differentiation (AD). We present the integration schemes that are automatically generated when differentiating the discretized state equation by AD. We analyze the convergence behavior of AD generated adjoints and show that the AD adjoints yield a discretized version of the adjoint of the discretize-then-optimize approach.

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