A MHD problem on unbounded domains - coupling of FEM and BEM

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The Maxwell equations describe processes in the elektromagnetic context. From this one can derive the following magnetohydrodynamic (MHD) problem

$$\begin{aligned} \partial_t \mathbf{B} &= -\nabla \times \mathbf{E} && \text{in } \Omega_c \\ \nabla \times \frac{1}{\mu} \mathbf{B} &= \begin{cases} \sigma \left(\mathbf{E} + \mathbf{u} \times \mathbf{B} + j^c \right) & \text{in } \Omega_c \\ 0 & \text{in } \Omega_v \end{cases} \\ \nabla \cdot \mathbf{B} &= 0 && \text{in } \Omega. \end{aligned}$$

In the so-called direct problem, the magnetic induction $\mathbf{B}$ and the electric field $\mathbf{E}$ are unknown and $\mathbf{u}$ is a given incompressible flow field. The domain Ω consists of conducting regions Ω_c (e.g. the Earth core) and insulating regions Ω_v (e.g. vacuum). After semidisrectization in time with the implicit Euler-scheme, a Lagrange finite element approach is used in the bounded region Ω_c and bounded regions of Ω_v. A boundary element approach is used in the unbounded insulating region of Ω_v following ideas of Kuhn/Steinbach (Math. Meths. Appl. Sc. 25 (2002), 357-371). We present results on the well-posedness of the continuous problem and for the semidiscrete coupled problem arising within each time step. Then we discuss algorithmic aspects for the solution of the coupled problem. Finally we present first results of the numerical analysis based on previous results in Chan er.al. (SINUM 44 (2006) 5, 1877-1902) and Guermond et al. (JCP 221 (2007) 1, 349-369).

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