Regularization results and error estimates for optimal control problems with sparsity functional

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A recent research area in optimization are nonsmooth problems which lead to sparse solutions. For example, this approach is successfully used in image processing and signal compression. The talk deals with optimal control problems featuring a nonsmooth term in the objective function which causes sparse solutions, i.e., solutions which vanish on parts of the domain. The problems are regularized to permit the use of the semi-smooth Newton method and read as

$$\text{Minimize} \quad J(y,u) = \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\alpha}{2}\|u\|^2_{L^2(\Omega)} + \beta\|u\|_{L^1(\Omega)}$$

$$\text{s.t.} \quad \begin{cases} Ay = u & \text{in } \Omega \\ y = 0 & \text{on } \partial\Omega \\ u_a \le u \le u_b & \text{in } \Omega. \end{cases}$$

Error estimates with respect to the regularization parameter α are provided and convergence as $\alpha \to 0$ is obtained. Moreover, finite element approximations are studied. A-priori as well as a-posteriori error estimates are developed and an adaptive method is used to discretize the problem. The theory is confirmed by numerical results which will be presented.

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