3D contact problems with given friction and a coefficient of friction depending on the solution

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In the talk we present the analysis, approximation and numerical realization of a mathematical model describing 3D contact problems for an elastic body unilaterally supported by a rigid foundation. On the common surface we take into account friction obeying the so-called Tresca model in which a threshold slip is given a priori. In the classical Tresca model the threshold slip is expressed as the product $\mathcal{F}g$, where g is a non-negative function and $\mathcal{F}$ is a coefficient of friction which does not depend on the solution. In some problems, however, $\mathcal{F}$ can be of the form $\mathcal{F} := \mathcal{F}(\|\boldsymbol{u}_t\|)$, i.e. the coefficient of friction depends on the magnitude of the tangential contact displacement. This contribution deals just with this case.

The weak formulation of our problem leads to an implicit variational inequality of elliptic type. To overcome difficulties related to this problem we characterize its solutions equivalently as fixed points of a mapping acting on the trace space defined on the contact part. Using this reformulation one obtains the existence of at least one weak solution to the problem for $\mathcal{F}$ represented by a continuous, positive and bounded function. Moreover, the solution is unique provided that $\mathcal{F}$ is Lipschitz continuous with a sufficiently small Lipschitz constant.

The method of successive approximations is proposed for finding fixed points. Each step of this method is defined by a contact problem with given friction in which the coefficient of friction does not depend on the solution. We introduce a mixed variational formulation of this problem in terms of displacements and contact stresses. This formulation is then discretized: the displacements are approximated by piecewise linear functions, the contact stresses by piecewise constant functions. Next, the dual formulation in terms of the discretized contact stresses is established. Finally, results of several numerical experiments are shown.

References:

[1] J. Haslinger, T. Ligurský: Approximation and numerical realization of 3D contact problems with given friction and a coefficient of friction depending on the solution. Submitted to Appl. Math., Praha, 2007.

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