Space-time discretizations for semilinear evolution problems

Miloslav Vlasak[1] Vit Dolejsi[2]

We deal with a numerical solution of a scalar non-stationary semilinear convection-diffusion equation, which represents a simplified model problem for the solution of the system of the compressible Navier-Stokes equations. The space semi-discretization is carried out with the aid of the discontinuous Galerkin finite element method (DGFEM), which is based on a piecewise discontinuous polynomial approximation.

The resulting system of the ordinary differential equations is often discretized by the explicit Runge-Kutta methods since these schemes have a high order of accuracy and they are simple for implementation. Their drawback is a strong restriction to the length of the time step. In order to avoid this disadvantage it is suitable to use an implicit time discretization, but a fully implicit scheme leads to a necessity to solve a nonlinear system of algebraic equations at each time step which is rather expensive.

Therefore, we develop a higher order unconditionally stable (or with a large domain of stability) time discretization technique which does not require a solution of nonlinear algebraic problem at each time step.

We analyse several approaches and derive a priori error estimates in the discrete analogues of the $L^\infty(0,T;L^2(\Omega))$-norm and the $L^2(0,T;H^1(\Omega))$-seminorm.

[1]Charles University Prague, Faculty of Mathematics and Physics, Sokolovska 83, 18675 Prague, Czech Republic,
vlasakmm@volny.cz
[2]Charles University Prague, Faculty of Mathematics and Physics, Sokolovska 83, 18675 Prague, Czech Republic,
dolejsi@karlin.mff.cuni.cz