Finite elements on stationary or evolving surfaces

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This lecture is about the numerical solution of partial differential equations on stationary or moving surfaces with the finite element method. PDEs on surfaces occur in many applications. For example they traditionally arise in materials science and fluid dynamics and more recently in biomechanics and in the mathematics of images.

The numerical method for the solution of surface PDEs depends on the representation of the surface. For parametric and for implicitly given surfaces finite element methods will be derived for linear model problems. An introduction into the basic surface finite element method for moving surfaces (ESFEM) is given. The model problem is the transport and diffusion of a material quantity on the surface. It will also be shown how an adequate mathematical formulation leads to a discretization in time which is efficient and easy to implement. Stability and convergence for the fully discretized PDE will be discussed in some detail.

As an important application the discretization of the classical bending energy (Willmore functional, Helfrich energy) will be presented.

Most of the work was done in collaboration with C. M. Elliott (Warwick).

References:

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