Mixed finite elements for thin elastic structures

Astrid Sinwel[1] Joachim Schöberl[2]

In this talk, we are concerned with solving the equations of linear elasticity on an anisotropic, tensor product domain $\Omega = \Omega_x \times \Omega_z$. There, $\Omega_x \subset \mathbb{R}^2$ is supposed to be a connected, Lipschitzian domain, whereas $\Omega_z = (0, d_z)$ corresponds to the thickness direction.

We derive a mixed variational formulation for the equations of linear elasticity

$$\begin{aligned} -\mathrm{div}(\sigma) &= f, \\ A\sigma &= \varepsilon(u). \end{aligned}$$

Here u denotes the displacement field, and σ the symmetric stress tensor. The linearized strain tensor $\varepsilon(u) = \frac{1}{2}(\nabla u + \nabla u^T)$ is given by the symmetric part of the gradient.

We choose $u \in H(\mathrm{curl})$, which implies tangential continuity of the displacement field. Starting from shape-regular, simplicial meshes for Ω_x, Ω_z we discretize the displacement field using Nédélec finite elements on the resulting, anisotropic tensor product mesh. For the stresses σ, this implies the necessity to have symmetric, tensor valued finite elements, where the normal normal component $\sigma_{nn} = n^T \sigma n$ is continuous across interfaces. We already developed conforming finite elements for simplicial meshes. As an extension, we propose a family of such elements of arbitrary order on the tensor product mesh.

Our elements are suitable for the discretization of beams or shells, as they do not suffer from shear locking. We see that the optimal order error estimates for the solution still hold true in the case of thin structures. We can show that the error depends on the anisotropic mesh sizes h_x, h_z, but not on their ratio. This is supported by our numerical results.

[1] Austrian Academy of Sciences, Radon Institute, Altenbergerstr. 69, 4040 Linz, Austria, astrid.sinwel@ricam.oeaw.ac.at

[2] Center for Computational Engineering Science, RWTH Aachen University, Pauwelsstrasse 19, 52074 Aachen, Germany, joachim.schoeberl@rwth-aachen.de