TECHNISCHE UNIVERSITÄT CHEMNITZ

Fakultät für Mathematik

Chemnitz FEM-Symposium 2008

Programme

Collection of abstracts

List of participants

Chemnitz, September 22 - 24, 2008

Scientific topics:

The symposium is devoted to all aspects of finite elements and wavelet methods in partial differential equations.

The topics include (but are not limited to)

- adaptive methods,
- parallel implementation,
- high order methods.

This year we particularly encourage talks on

- Finite Elements in Biomechanics and Biomedicine
- Shells and Plates
- PDEs on surfaces

Invited Speakers:

Gerhard A. Holzapfel (Graz)

Luisa Donatella Marini (Pavia)

Gerhard Dziuk (Freiburg)

Scientific Committee:

Th. Apel (München), G. Haase (Graz), H. Harbrecht (Bonn),
B. Heinrich (Chemnitz), M. Jung (Dresden), U. Langer (Linz),
A. Meyer (Chemnitz), A. Rösch (Duisburg), O. Steinbach (Graz)

WWW: http://www.tu-chemnitz.de/mathematik/fem-symposium/

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Chemnitz, September 22 - 24, 2008

Programme for Monday, September 22, 2008

9:00 A. Meyer
Welcome

Plates and Shells
Chairman: A. Meyer
Room: "Lichtenwalde"

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A posteriori estimates for Discontinuous Galerkin methods for second order elliptic problems

9:55 L.E. Alaoui 7
A finite element analysis of a penalized Koiter model

10:20 A. Sinwel 46
Mixed finite elements for thin elastic structures

10:45 *Tea and coffee break*

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12:00	P. Gruber .... 20 A New Approach for Solving and Analyzing Problems in Elastoplasticity	K. Krumbiegel .... 28 Finite element error analysis of a state-constrained optimal control problem with boundary control
12:25	A. Meyer .... 33 Residual Based Error Estimators for Deformation Problems with Large Strain	M. Bernauer .... 8 Towards the Optimal Control of a One-Phase Stefan Problem

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12:50 *Lunch break*

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15:20	C. Pechstein . . . . 36 A rigorous analysis of FETI methods for high-contrast coefficients not resolved by the subdomain partitioning	G. Matthies . . . . 32 The one-level approach of the local projection method applied to inf-sup stable discretisations of the Oseen problem
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17:20	M. Huber . . . . 24 Hybrid finite element methods for time harmonic Maxwell's equations	M. Růžička . . . . 40 Finite element simulation of interaction of flow and an airfoil with three degrees of freedom
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FE in Biomechanics
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12:15	A. Meyer Closing
12:30	*Lunch*

A finite element analysis of a penalized Koiter model

Linda El Alaoui[1] Adel Blouza[2] Aouadi-Mani Saloua[3]

We present a penalized version of Koiter model for linearly elastic shells with little regularity. This model is discretized by a conforming finite element method for which a convergence result holds. We derive residual a posteriori error estimators leading to upper and lower bounds of the discretization error. Finally, we present numerical results validating our method and showing the efficiency of the error estimators.

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Towards the Optimal Control of a One-Phase Stefan Problem

Martin Bernauer [1]

In this talk, we discuss a motion planning problem for a one-phase Stefan problem. This problem class is modeled by a parabolic PDE on a domain with moving boundary. For capturing the moving boundary, level-set techniques are used. In the first part, we present a solver for the direct problem that is based on a discontinuous Galerkin scheme for the level-set equation and the X-FEM method for the approximation of the temperature field. We illustrate the behavior of this solver with a numerical example. In the second part of the talk, optimality conditions for a motion planning problem for a one-phase Stefan problem in level-set formulation will be discussed. The derivation of these techniques is based on optimal shape design tools.

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Multilevel solvers for hp-FEM discretizations in 3D using hexahedral elements

Sven Beuchler[1]

In this talk we investigate the discretization of an elliptic boundary value problem in 3D by means of the hp-version of the finite element method using a mesh of hexahedrons. The corresponding linear system is solved by a preconditioned conjugate gradient method. The construction of the preconditioner is based on an inexact additive overlapping Schwarz method which was suggested by Pavarino.

The remaining subproblems are treated by a tensor product based preconditioner. This preconditioner uses a basis transformation into a basis which is stable in L_2 and H^1. The construction is based on interpretations of the p-FEM mass and stiffness matrix as weighted h-FEM matrices and a simultaneous diagonalization of these matrices using wavelets.

Several numerical examples show the efficiency of the proposed method.

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Finite element analysis of an unilateral contact problem

Adel Blouza [1]

We perform the analysis of the unilateral contact problem between two elastic membranes. We propose a finite element descretization of the corresponding system of variational inequalities and prove its well posedeness. We also establish a priori error estimates.

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Parallel simulator of saturated porous media with fractures

Jan Březina[1]

Motivated by numerous applications in the hydrogeology, we develop a simulator of the underground water flow in a disrupted rock massif. The small cracks are represented by a virtual water conductivity of the rock, while the large cracks and their intersections are modeled by an overlapping mesh of surfaces and lines respectively. For the discretization of the 3D-2D-1D problem of the water flow we use a hybridization of a mixed finite elements in order to obtain a divergence free approximation of the velocity field. The simulator allows nonconforming discretization of the individual dimensions, but we lack a theoretical justification for this approach. The parallelization of the simulator is done via the domain decomposition using essentially the PETSC library.

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Finite element simulation of interaction of compressible flow and a moving airfoil

Jan Cesenek[1] Miloslav Feistauer[2]

This paper deals with the formulation of a numerical scheme for solving compressible flow past moving bodies. We use the discontinuos Galerkin finite element method for the space semi-discretization and the Euler backward formula for the time discretization. Moreover, we use ALE mapping for the treatment of a time depended domain. We shall present computational results obtained for the flow past a vibrating airfoil.

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Discontinuous Galerkin approximations for the Navier-Stokes equations

Konstantinos Chrysafinos[1]

We will discuss numerical schemes for the evolutionary Navier-Stokes equations. The schemes considered here, are discontinuous in time and conforming in space and of arbitrarily high-order. Fully-discrete error estimates are presented in both 2d-3d cases, and the viscosity constant is carefully tracked. The estimates are derived under low regularity assumptions. The main result states that the errors are bounded by projection errors of the exact solution which exhibit optimal rates when the solutions are smooth. The key ingredient of the proof is based on the development of "symmetric" error estimates for a parabolic analog of the classical Stokes projection, and it is motivated on the recent work on the convection-diffusion equation [1].

Joint work with Noel J. Walkington, Department of Mathematics, CMU, USA.

References:

[1] 1. K. Chrysafinos and N. J. Walkington, Lagrangian and moving mesh methods for the convection-diffusion equation, ESAIM M2AN, 42, 2008, pp 27-56.

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hp-adaptive Discontinuous Galerkin method for convection dominated evolution equations based on an a-posteriori error estimate

Andreas Dedner[1]

In this talk we present an hp-adaptive scheme in space and time for the discretization of a non-linear system of evolution equations

$$\partial_t U(t,x) + \nabla \cdot \big(F(U(t,x),t,x) + a(U(t,x),t,x)\nabla U(t,x)\big) = S(U(t,x),t,x) \ .$$

We base our method on the higher order Discontinuous Galerkin method in space and explicit multistep methods in time. We use h-adaptivity, i.e., general grid structures with non-conform adaptivity and local time stepping, to achieve a high degree of efficiency. Since our focus is on the convection dominated case, we discuss approaches for gradient limiting and p-adaptivity for stabilizing the scheme in the regions of strong gradients or discontinueties. The basis of our scheme is an a-posteriori error estimate for the semi-discrete method which will be briefly discussed and compared with a heuristic approach. For the implementation of the scheme we use the software environment DUNE.

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FEM/BEM-coupling for fluid structure interaction in 2D and 3D

Catalina Dominguez[1] M. Maischak E.P. Stephan

We consider time-harmonic vibration and scattering problems for inhomogeneous, isotropic, elastic solids surrounded by a compressible, inviscid, homogeneous fluid. For such problems a finite element/ boundary element coupling formulation is given in [1]. The validity of the method for all wave numbers is ensured by a boundary integral equation (see [3]). [1] and [2] show existence and uniqueness of the solution, assuming a smooth domain interface. Based on the concept of strong ellipticity as in [4,5], quasi-optimal convergence of the method is shown. The method is implemented for the 2D and 3D case. Error estimators and fast solvers are investigated and numerical experiments are presented.

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[3] Kress; Minimizing the condition number of boundary integral operators in acoustic and electromagnetic scattering; Quarterly of applied mathematics, Vol. 38, pp. 323-341, (1985)
[4] S. Hildebrandt, E. Weinholz: Construction proofs of representation theorems in separable Hilbert space, Comm. Pure Appl. Math. 17, pag. 369-373, (1964).
[5] R. C. MacCamy, E. P. Stephan: A boundary element method for an exterior problem for three-dimensional Maxwell's equations. Applicable Anal. 16, no. 2, pag. 141-163, (1964).

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Finite elements on stationary or evolving surfaces

Gerhard Dziuk[1]

This lecture is about the numerical solution of partial differential equations on stationary or moving surfaces with the finite element method. PDEs on surfaces occur in many applications. For example they traditionally arise in materials science and fluid dynamics and more recently in biomechanics and in the mathematics of images.

The numerical method for the solution of surface PDEs depends on the representation of the surface. For parametric and for implicitly given surfaces finite element methods will be derived for linear model problems. An introduction into the basic surface finite element method for moving surfaces (ESFEM) is given. The model problem is the transport and diffusion of a material quantity on the surface. It will also be shown how an adequate mathematical formulation leads to a discretization in time which is efficient and easy to implement. Stability and convergence for the fully discretized PDE will be discussed in some detail.

As an important application the discretization of the classical bending energy (Willmore functional, Helfrich energy) will be presented.

Most of the work was done in collaboration with C. M. Elliott (Warwick).

References:

[1] G. Dziuk, C. M. Elliott: Finite elements on evolving surfaces. IMA J. Numer. Anal. 27/2 (2006) 262–292.
[2] G. Dziuk, C. M. Elliott: An Eulerian approach to transport and diffusion on evolving implicit surfaces. To appear in Comp. Vis. Sci. DOI 10.1007/s00791-008-0122-0.
[3] G. Dziuk: Computational parametric Willmore flow. To appear in Numer. Math.

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Hybrid Finite Element Methods for Interface Problems

Herbert Egger[1]

We propose a family of hybrid finite element methods for interface problems with possibly non-matching meshes. As the a-priori error analysis shows, these methods are very robust with respect to the choice of spaces used for the discretization. The hybrid methods allow for assembling on the subdomain level, as well as for elimination of the interior unknowns on the subdomains, which are desired features of any domain decomposition method. We also try to clarify the relation to mixed and discontinuous Galerkin methods, as well as to Nitsche-type mortaring, and confirm our theoretical results by presenting numerical tests.

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An Unfitted Finite Element Method using Discontinuous Galerkin

Christian Engwer[1]

In the simulation of physical, biological and chemical processes one often has to deal with complex shaped domains. In such simulations a good approximation of the geometrical shape is crucial in order to obtain reliable numerical results. At the same time the interest often lies in a solution on a coarser scale, which would allow a smaller number of unknowns.

A new discretization scheme for solving PDEs in complex domains, e.g. on the pore scale, was developed. It combines the idea of Unfitted Finite Elements with a Discontinuous Galerkin (DG) Finite Elements discretization. The degrees of freedom are determined by the structured grid, while the fine structures of the domains shape are preserved by limiting the support of the shape functions to the intersection of the structured grid cells and the domain. This method allows the minimal number of unknowns to be independent of the shape of the domain, even if their size is significantly bigger than that of the structures in the shape of the domain.

In this talk an introduction into the Unfitted Discontinuous Galerkin Method will be given. Examples show the stability of the method even for pathologic cases. The method itself is easily applicable to different DG discretizations and different types of PDEs. Within this talk computations are shown for an elliptic model problem.

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Applying Unfitted Discontinuous Galerkin to Time Dependent Problems

Jorrit Fahlke[1]

Unfitted Discontinuous Galerkin (UDG) is a new discretization scheme for problems in complex domains. It was developed for elliptic PDEs as a combination of Unfitted Finite Elements and Discontinuous Galerkin methods. It allows a relatively small number of unknowns while still providing a good approximation of the geometrical shape of the domain.

In this presentation we apply UDG to time dependent problems in complex domains. We will show the discretization for the instationary convection-diffusion problem. Numerical results for an artificial complex domain and second order are shown. Full order of convergence is obtained for the space as well as for the time discretization.

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A New Approach for Solving and Analyzing Problems in Elastoplasticity

Peter Gruber[1]

Elastoplastic deformation problems can be modelled as the minimization of a convex energy functional with respect to the displacement field. Such functional has a first, but not a second derivative. For minimization a Newton-like method is applied where the missing second derivative is replaced by a slanting function – a concept which was introduced in the year 2000 by X. Chen, Z. Nashed, and L. Qi. In this talk theoretical convergence results and numerical experiments are presented.

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GPU Accelerated Algebraic Multigrid

Gundolf Haase[1] Manfred Liebmann[2] Gernot Plank[3]

The race for higher and higher clock frequencies hit a physical road block five years ago, forcing the big processor companies to focus on multi–core architectures instead. One major direction in semiconductor industry is the many–core GPU architecture of Nvidia and AMD with hundreds of simple scalar processors with only a small amount of on–chip memory and a very high bandwidth memory interface. The current generation of GPUs deliver 1TFLOPS peak in single precision arithmetic and 100 GFLOPS in double precision. The high performance scalar processors on the GPU are fed by a very high bandwidth memory interface to the on-board DRAM with up to 150GB/s throughput. This is more than one order of magnitude faster than on a typical CPU based server.

The easy–to–use to CUDA-interface for programming NVidia cards encouraged us to check whether many–core GPUs are useful in scientific computing for solving systems of equations with sparse unstructured matrices from 3D applications. First results on the NVIDIA Geforce 8800 GT showed an acceleration of 10 in comparison to a high-end CPU for the sparse matrix–vector product in single precision. Based on this matrix-vector product we accelerated our Algebraic Multigrid (AMG) solver also by a factor of 10 on the GPU. The recent tests on a pre–release of the NVIDIA GTX 280 confirmed the measured accelerations also for double precision arithmetics. This allows together with our modular parallelization toolbox a very fast and efficient parallel AMG solver on a rather cheap parallel computer based on GPUs from the shelf.

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A finite element method for PDEs with stochastic input data

Helmut Harbrecht[1] Reinhold Schneider[2] Christoph Schwab[3]

We compute the expectation and the two-point correlation of the solution to elliptic boundary value problems with stochastic input data. Besides stochastic loadings, via perturbation theory, our approach covers also elliptic problems on stochastic domains or with stochastic coefficients. The solution's two-point correlation satisfies a deterministic boundary value problem with the two-fold tensor product operator on the two-fold tensor tensor product domain. For its numerical solution we apply a sparse tensor product approximation by multilevel frames. This way standard finite element techniques can be used. Numerical examples illustrate feasibility and scope of the method.

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Numerical treatments of strong discontinuities within soft tissue biomechanics: state of the art and challenges ahead

Gerhard A. Holzapfel[1]

Several soft biological tissues such as arteries in health and disease typically show highly nonlinear and volume-preserving mechanical responses requiring sophisticated numerical approaches [1]. Multi-field variational principles are frequently used to solve related boundary-value problems leading to mixed or hybrid methods for finite elements. In addition, collagen is the ubiquitous load-bearing and reinforcing protein in many soft and hard biological tissues, which forms an important structural basis. The structural arrangement of collagen leads to the characteristic anisotropic behavior of the material which challenges the related constitutive formulation [2].

Tissue dissection is one form of trauma involving laceration and/or cleavage of the tissue. For capturing the jump in the displacement field for a dissection a traditional kinematic concept is not sufficient; we need to involve the kinematics of strong discontinuities [3]. This talk presents the latest experimental findings of soft tissue dissections, and associated material and finite element modeling. Future multi-disciplinary challenges that we will face the next decade will be discussed.

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[3] T.C. Gasser and G.A. Holzapfel. Modeling the propagation of arterial dissection. Eur. J. Mech. A/Solids 25:617-633, 2006.

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Hybrid finite element methods for time harmonic Maxwell's equations

Martin Huber[1] Joachim Schöberl[2] Antti Hannukainen[3]

The topic of this talk is the solution of the time harmonic Maxwell's equations for high wave numbers with high order finite elements. We use discrete eigenfunctions as basis for the finite element spaces and therfore we can efficiently work with polynomial orders up to thousands. This allows us to resolve high frequent waves even on very coarse meshes. For structured rectangular grids the 2D eigenvalue problem decouples into two 1D problems. We compare two formulations of the problem: a primal hybrid- and a mixed hybrid formulation. Finally we present numerical examples to demonstrate the effectiveness of our approach.

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Reduced Basis Method for Quadratically Nonlinear Transport Equations

Nadine Jung[1] B. Haasdonk[2] D. Kroener[3]

With the Reduced Basis Method (RBM) we can solve a given parametrized PDE for many parameters which are not known in advance faster then by Finite Element Method (FEM). Initially we choose in a suitable way N parameters $\mu_1, ..., \mu_N$ within a certain interval and compute the FE solution $u_H(\mu_i)$ for $1 \leq i \leq N$ on an extremely fine mesh. The number N is small compared to the dimension of the FE space. We construct a new space X_N, spanned by these N FE approximations. Then, in order to compute approximations for many different parameters μ, different from $\mu_1, ..., \mu_N$, we compute an RB approximation $u_N(\mu)$ in the lower dimensional space X_N. Because of the lower dimensional space X_N the computations of the RB approximation $u_N(\mu)$ is faster than the computation of the FE approximation $u_H(\mu)$. In [2] the RBM is applied to the stationary, viscous Burgers equation. [2] contains an a posteriori error estimator between $u_N(\mu)$ and $u_H(\mu)$ as well as an existence proof. The results of [2] are extended in [1] to a wider class of parameter choices and the instationary viscous Burgers equation, i.e. quadratically nonlinear transport equations.

The a posteriori estimator between the RB approximation $u_N(\mu)$ and FE approximation $u_H(\mu)$ justifies the approach and allows a more efficient basis construction.

Numerical experiments on a parameter-dependent transport problem, discretized with backward Euler, Newton Method and FEM, demonstrate the applicability of the model reduction technique. We compare the results of the RBM with FEM. The CPU times demonstrate the efficiency, in particular we gain a factor $\frac{1}{66}$ for the CPU time.

References:

[1] N. Jung, *Anwendung der Reduzierten Basis Methode auf quadratisch instationäre Transportgleichungen - die instationäre viskose Burgers Gleichung*, University of Freiburg, Diplomarbeit, University of Freiburg, May 2008

[2] K. Veroy, and A.T. Patera, *Certified real-time solution of the parametrized steady incompressible Navier-Stokes equations: rigorous reduced-basis a posteriori error bounds.*, Int. J. Numer. Methods Fluids **47** (2005), 8-9, pp. 773-788.

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A variational multiscale method for turbulent flow simulations with an adaptive choice of the large scale space

Adela Kindl[1] Volker John[2]

Variational Multiscale (VMS) methods are a rather new approach in simulating turbulent flows. The basic idea of VMS methods, in contrast to traditional LES, is the use of variational projections instead of filtering for the scale decomposition, thus eliminating several difficulties of the traditional LES, e.g. commutation errors.

We consider a three scale decomposition of the flow: large (resolved) scales, resolved small scales, and (small) unresolved scales. Assuming that the direct influence of the unresolved scales onto the large scales is negligible, and thus the direct influence of the unresolved scales is confined to the resolved small scales, the influence of the unresolved scales onto the resolved small scales is modeled with a turbulence model. In the context of finite element discretizations, the parameters of a VMS method are the finite element spaces used to define the scale decomposition and the turbulence model acting directly only on the small scales. Regarding the turbulence model, the parameter in the additional viscous term added to the small scale equation is generally chosen to be an eddy viscosity model of Smagorinsky type. Regarding the spaces standard finite element spaces are considered for all resolved scales and the separation of the large and the resolved small scales is achieved through an additional large scale space.

We will present and discuss an adaptive choice of the large scale space, allowing different polynomial degrees on different mesh cells. This so called adaptive large scale projection-based method (ALS-VMS), aims at avoiding the too large diffusivity of the Smagorinsky model in regions with little turbulence, and this through an appropriate choice of the large scale space.

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On the stability of finite element discretizations of convection-diffusion-reaction equations

Petr Knobloch[1] Lutz Tobiska

A priori error estimates for the local projection stabilization applied to convection-diffusion-reaction equations are generally based on the coercivity of the underlying bilinear form with respect to the local projection norm. We show that the bilinear form of the local projection stabilization satisfies an inf-sup condition in a stronger norm which is equivalent to that of the streamline upwind/Petrov-Galerkin method. As a consequence we get some insight in the stabilization mechanism of Galerkin discretizations of higher order.

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Finite element error analysis of a state-constrained optimal control problem with boundary control

Klaus Krumbiegel[1] Meyer, Christian[2] Rösch, Arnd[3]

We consider a linear quadratic optimal control problem with pointwise state constraints and control constraints, where the control acts at the boundary. It is well known that problems with pointwise state constraints inhibit a lot of difficulties since the Lagrange multipliers are in general only Borel measures. Therefore, different regularization concepts are developed in the last years. However, a direct extension of the Lavrentiev regularization concept is not possible since the control is not defined in the domain where the state constraints are given.
We will use the concept of a virtual distributed control in the domain Ω. Thus the Lavrentiev regularization is applicable. The effect of regularization is influenced by different parameter functions depending on a regularization parameter $\varepsilon > 0$. Furthermore, the problem is discretized by finite elements. We derive an error estimate of the optimal solution of the original problem to the corresponding discretized and regularized one. Since the regularization error and the discretization error appears simultaneously, we have to balance the regularization parameter and the mesh size in an appropriate manner.

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3D contact problems with given friction and a coefficient of friction depending on the solution

Tomáš Ligurský[1] Jaroslav Haslinger[2]

In the talk we present the analysis, approximation and numerical realization of a mathematical model describing 3D contact problems for an elastic body unilaterally supported by a rigid foundation. On the common surface we take into account friction obeying the so-called Tresca model in which a threshold slip is given a priori. In the classical Tresca model the threshold slip is expressed as the product $\mathcal{F}g$, where g is a non-negative function and $\mathcal{F}$ is a coefficient of friction which does not depend on the solution. In some problems, however, $\mathcal{F}$ can be of the form $\mathcal{F} := \mathcal{F}(\|\boldsymbol{u}_t\|)$, i.e. the coefficient of friction depends on the magnitude of the tangential contact displacement. This contribution deals just with this case.

The weak formulation of our problem leads to an implicit variational inequality of elliptic type. To overcome difficulties related to this problem we characterize its solutions equivalently as fixed points of a mapping acting on the trace space defined on the contact part. Using this reformulation one obtains the existence of at least one weak solution to the problem for $\mathcal{F}$ represented by a continuous, positive and bounded function. Moreover, the solution is unique provided that $\mathcal{F}$ is Lipschitz continuous with a sufficiently small Lipschitz constant.

The method of successive approximations is proposed for finding fixed points. Each step of this method is defined by a contact problem with given friction in which the coefficient of friction does not depend on the solution. We introduce a mixed variational formulation of this problem in terms of displacements and contact stresses. This formulation is then discretized: the displacements are approximated by piecewise linear functions, the contact stresses by piecewise constant functions. Next, the dual formulation in terms of the discretized contact stresses is established. Finally, results of several numerical experiments are shown.

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Optimal control of singularly perturbed advection-diffusion-reaction problems with control constraints

Gert Lube[1] B. Tews

In this talk, we consider the numerical analysis of discretized optimal control problems governed by a linear advection-diffusion-reaction equation with pointwise control constraints. The standard Galerkin discretization is stabilized via the local projection approach which leads to a symmetric optimality system at the discrete level. The optimal control problem simultaneously covers distributed and Robin boundary control. In the singularly perturbed case, the boundary control at characteristic parts of the boundary can be seen as regularization of a Dirichlet boundary control. Some numerical tests confirm the analytical results.

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A posteriori estimates for Discontinuous Galerkin methods for second order elliptic problems

L. Donatella Marini[1]

We analyze Discontinuous Galerkin formulations for second order elliptic problems in mixed form. Using the "weighted residual" approach of [1] we carry out the a posteriori analysis in an abstract framework, without specifying the choice of the weighting operators. In such a way we identify the minimal approximation properties required on the operators to guarantee lower and upper bounds for the energy norm of the error. We then show that this unified approach applies to all the Discontinuous Galerkin schemes presented so far in the literature.

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The one-level approach of the local projection method applied to inf-sup stable discretisations of the Oseen problem

Gunar Matthies[1] Lutz Tobiska[2]

The Oseen problem occurs as an important subproblem during the iterative solution of the stationary and instationary Navier–Stokes equations.

The standard Galerkin finite element method can be applied after choosing approximation spaces for velocity and pressure. If these spaces satisfy an inf-sup condition, no pressure stabilisation is needed. However, the Galerkin method suffers in general from spurious oscillations in the velocity which are caused by the dominating convection.

To handle this instability, the local projection stabilisation will be used. Originally, the local projection technique was proposed as a two-level method where the projection space is defined on a coarser mesh. Unfortunately, this approach leads to an increased discretisation stencil.

Our main objective is to analyse the convergence properties of the one-level approach of the local projection stabilisation applied to inf-sup stable discretisations of the Oseen problem. Moreover, we propose new inf-sup stable finite element pairs approximating both velocity and pressure by elements of order r. In contrast to the ‘classical’ equal order interpolation, the velocity components and the pressure are discretised by different elements. In the convection dominated case $\nu < h$, we show for these pairs of finite element spaces an error estimate of order $r + 1/2$.

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Residual Based Error Estimators for Deformation Problems with Large Strain

Arnd Meyer[1]

After introducing basic geometry description for "large deformations" we arive at the main non-linear weak formulation of the problem as integrals over the initial (given undeformed) domain. With the most simple material law of linear elasticity we present first results of adaptive FE-simulations. Here, the question of appropriate error estimators for defining the correct mesh control is discussed.

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A two step time discretization of Willmore flow

Nadine Olischläger[1] Martin Rumpf[2]

In surface restoration a damaged region of a surface is replaced by a surface patch which restores the region in a suitable way. In particular one aims for a $C1$-continuity at the patch boundary. The $L2$-gradient flow of the Willmore functional is considered to measure fairness and to allow appropriate boundary conditions ensuring continuity of the normal field. Often, real world restoration problems are of anisotropic nature, e.g., if an edge or a corner of a surface is destroyed. The generalization of the classical Willmore functional is a fourth order energy having Wulff shapes as the only minimizers. The corresponding $L2$-gradient flow as the actual restoration process leads to a system of fourth order partial differential equation. Usual discretizations approach for this PDE require the evaluation of higher order derivatives of the anistropy and come along with strong restrictions on the time step. To overcome these difficulties we develop a two step time discretization of the Willmore flow. It is based on abstract approaches for gradient flows which balance energy decay and dissipation encoded in the metric. In a variational approach for the time discrete Willmore flow we have to solve a nested variational problem in each time step. The inner minimization problem represents a time step of mean curvature motion on the unknown surface at the next time step. The discrete speed of propagation from this scheme is considered as a suitable approximation of the mean curvature vector. Hence, this vector is incorporated in the energy term of the outer minimization problem for a time step of Willmore flow. Due to the built-in regularized fully implicit Willmore functional the resulting scheme turned out to be very robust in numerical experiments and time steps of the order of the spatial grid size are feasible.

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Transient Boundary Element Method and Numerical Evaluation of Retarded Potentials

Elke Ostermann[1] Ernst P. Stephan[2] M. Maischak[3]

We will present our latest results concerning the modeling of transient wave propagation using the boundary element method. The special structure of the fundamental solution of the wave equation leads to a close interaction of space and time variables in a so-called retarded time-argument. The corresponding retarded potentials lead naturally to sparse matrices in contrast to the dense matrices usually associated with the BEM, but in each time step the matrix has to be stored, creating a history of matrices. The sparsity of these matrices is a result of the intersection of the domain of influence of an element with the boundary domain, such that the actual number of interacting elements is rather small. In this talk we discuss non-nearfield-like singularities of the retarded potentials and their consequence for the numerical evaluation of the Galerkin integrals.

Moreover, we present stable numerical simulations for the transient sound radiation in three dimensions using the developed quadrature scheme.

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A rigorous analysis of FETI methods for high-contrast coefficients not resolved by the subdomain partitioning

Clemens Pechstein[1] Robert Scheichl[2]

Finite element tearing and interconnecting (FETI) methods are highly efficient parallel iterative substructuring solvers, i. e., non-overlapping domain decomposition methods. In the scalar elliptic case $-\nabla \cdot [\alpha \nabla u] = f$, it is known that the condition number of the preconditioned system is bounded by $C\,(1 + \log(H/h))^2$, where h is the mesh parameter, H is the subdomain diameter, and the constant C is independent of h, H, and the number of subdomains. In the case that the coefficient $\alpha(\cdot)$ is constant on each subdomain, the constant C in the estimate is independent of $\alpha(\cdot)$, i. e., the method is robust with respect to large jumps across subdomain interfaces. In the general case, however, a straightfoward estimate delivers $C \lesssim \max_i \max_{x,y \in \Omega_i} \frac{\alpha(x)}{\alpha(y)}$, where the Ω_i are the subdomains.

As observed numerically by several authors (e. g., [Rixen, Farhat, IJNME 44, 1999], [Klawonn, Rheinbach, CMAME 196, 2007], [Langer, Pechstein, ZAMM 86, 2006], simple generalizations of FETI-type method are still robust even in case of large coefficient variation inside the subdomains. In this talk we present a rigorous analysis for a class of non-trivial coefficient distributions. In particular we can allow for arbitrary coefficient values in the subdomain interiors, and two different materials with moderate coefficient variation in the remainder of each subdomain. Our bounds are independent of jump sizes but essentially depend on the geometry of the materials. In particular we can prove the robustness observed by Rixen and Farhat. The first author has been supported by the Austrian Science Founds (FWF) under grant F1306.

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Finite element solution of compressible flow in time-dependent domains

Jaroslava Prokopová[1] Miloslav Feistauer[2] Václav Kučera

The paper is concerned with the modelling of inviscid, compressible flow in time-dependent domains. This problem plays an important role in several areas of research, as for example, flow past an oscillating airfoil or flow in human vocal folds. For investigation of such type of flow we consider the system of the Euler equations describing the motion of compressible flow. The time-dependence of the computational domain is taken into account with the aid of the Arbitrary Eulerian-Lagrangian method. Using this method, we derive two different formulations of the Euler equations. The space discretization of the governing system in the ALE formulation is carried out by the discontinuous Galerkin finite element method. For the time discretization a semi-implicit technique is developed, which is unconditionally stable and allows the solution of compressible flow with a wide range of Mach numbers. Numerical experiments carried out for the case of flow through a channel with a moving wall show the applicability and robustness of the method.

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Hierarchical surface mesh generation for Wavelet BEM solvers

Randrianarivony Maharavo[1]

We report on our results about surface mesh generation from CAD models. Our method is featured by its ability of generating hierarchical meshes which are very useful for solvers requiring nested trial spaces. To construct wavelets on manifolds the parametric description of the boundary surface is needed.

We need to decompose the boundary of a solid into four-sided patches F_i such that there is a regular mapping γ_i from the unit square to each F_i. Since we use Coons functions to generate the mappings γ_i, all curves are parametrized in arc length so that the functions γ_i match well at surface joints. That result is valid for any blending functions of the Coons patches. We use a reparametrization approach which keeps the shape of the initial curves while achieving arc length parametrization.

The decomposition techniques are applied to real CAD data which come from IGES files. Comments about generalization into 3D solid meshes are provided.

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Anisotropic Splitting of A Posteriori Errors Estimated with the Dual Weighted Residual Method

Thomas Richter[1]

We will present the extension of the Dual Weighted Residual method (DWR) for a posteriori error estimation to anisotropic finite elements.

The common approach for anisotropic adaptive mesh control combines the DWR-method for determining regions of refinement with the recovery of higher derivatives for determining the optimal refinement direction. This two-step approach however does not yield the correct balancing of primal and adjoint anisotropy information in all cases.

Our work will allow for a direct estimation of directional errors with the DWR-method leading to a unified approach for mesh refinement and identification of anisotropy.

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Finite element simulation of interaction of flow and an airfoil with three degrees of freedom

Martin Růžička[1] Miloslav Feistauer[2] Petr Sváček[3]

The subject of the contribution is the numerical solution of the two-dimensional incompressible viscous flow and a vibrating airfoil. In our model the airfoil is represented by a structure, which can perform vertical and torsional vibrations and its flap flutters. Mathematical model of flow is formed by the system of two dimensional non-stationary Navier-Stokes equations and continuity equation, coupled by initial condition and mixed boundary conditions. Stabilized finite element method is used to get the numerical solution. With regard to the moving airfoil, the computational domain is time dependent. This involvement requires to use suitable technique for the simulation on moving computational grids. To solve this we apply the Arbitrary Lagrangian-Eulerian method. The numerical simulation consists of the coupling procedure of the computation of flow with the system of ordinary differential equations describing the airfoil motion. The ordinary differential equations for the airfoil are solved by an appropriate solver. In our case we employ the Runge-Kutta method. The finite elements method leads to a large discrete system of nonlinear algebraic equations. We solve nonlinearity by implementing the Oseen iterations. This method splits nonlinearity into a sequence of linear problems with solutions converging to the solution of the nonlinear Navier-Stokes problem. High Reynolds numbers up to 10^6 require the application of a suitable stabilization of the finite element discretization. Numerical tests prove that the developed method is sufficiently accurate and robust.

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Multidimensional Coupling in a Human Knee Model

Oliver Sander[1]

We present a new way to couple bones modelled as linear elastic three-dimensional objects to ligaments modelled as Cosserat rods. Starting from a full 3d nonlinear elastic formulation we derive suitable coupling conditions for the reduced model. These involve the total force and torque transmitted through the interface as well as its averaged position and an average orientation. The resulting domain decomposition problem is solved using a Dirichlet–Neumann algorithm.

The configuration space of a special Cosserat rod is the set of all continuous mappings from a given interval to $\mathbb{R}^3 \times SO(3)$. We introduce geodesic finite elements as a natural way to discretize problems in such a nonlinear space. For the minimization of the rod energy functional we present an ∞-norm Riemannian trust-region algorithm. This yields an efficient method with provable global convergence.

We use this coupling approach to model a human knee joint. The use of rods for the ligaments decreases the overall number of degrees of freedom and avoids meshing problems. The additional problem of modelling the contact between the bones is treated using a mortar element discretization and a truncated nonsmooth Newton multigrid method for the solution of the resulting discrete system.

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Finite Element Methods for Transient Convection-Diffusion-Reaction Equations

Ellen Schmeyer[1] Volker John[2]

An application of time–dependent convection–diffusion–reaction equations is the simulation of a chemical reaction in a flow field. Such reactions are modeled via a coupled system of nonlinear convection–diffusion–reaction equations for the concentrations of the reactants and the products. To simulate such a process, a method is needed that is on the one hand able to compute sharp layers and which on the other hand prevents the occurence of spurious oscillations. We have studied the Streamline–Upwind Petrov–Galerkin (SUPG) method for several parameters, different Spurious Oscillations at Layers Diminishing (SOLD) methods, a Local Projection Stabilization (LPS) scheme and two Finite Element Method Flux–Corrected–Transport (FEM–FCT) methods to identify methods which fulfil these requirements.

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Asymptotic expansions of highly conductive thin sheets

Kersten Schmidt[1] Sébastien Tordeux[2]

Sensitive measurement and control equipment are protected from disturbing electromagnetic fields by thin shielding sheets. Alternatively to discretisation of the sheets, the electromagnetic fields are modeled only in the surrounding of the layer taking them into account with the so called Generalised Impedance Boundary Conditions.

We study the shielding effect by means of the model problem of a diffusion equation with additional dissipation in the curved thin sheet. We use the asymptotic expansion techniques to derive a limit problem, when the thickness of the sheet ε tends to zero, as well as the models for contribution to the solution of higher order in ε. These problems are posed in limit area of vanishing ε with condition for the jump of the solution and it's normal derivative, which avoid to mesh the computational domain, even locally, at the scale of ε.

For smooth sheets we derive the problems for arbitrary order and show their existence and uniqueness. Numerical experiments for the problems up to second order show the asymptotic convergence of the solution of right order in mean of the thickness parameter ε.

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Finite element analysis of musculoskeletal loadings

Leo-Philipp Schmidt[1]

In vivo measurements of mechanical stresses in the knee joint are limited to the use of instrumented implants and therefore connected with severe surgical treatments. For this reason there is great interest in numerical simulations providing detailed information concerning the mechanical behavior of hard and soft tissues in different medical application areas.

Our goal is to get a better understanding for the key factors influencing the stress distribution in the knee joint. As large animals are standard models for the investigation of fundamental mechanical behaviors in this field of study our investigations are based on data concerning sheep locomotion. Investigations of the sensitivity concerning some of the involved mechanical parameters will lead to better insight into the complex interrelations in the knee joint.

In our knee model we use linear elastic bones modeled by simplicial grids. Adaptive refinement using parametrized boundaries permits satisfying approximation of the curved bone surfaces which are reconstructed from high resolution CT scans. Furthermore our model contains cruciate and collateral ligaments modeled by one-dimensional non-linear Cosserat rods. Coupling these mixed-dimensional elements drives into heterogeneous problems which are encountered using a Dirichlet-Neumann scheme. The underlying multi-body contact problems are solved by a multigrid method which combines a high degree of reliability and effectiveness.

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Goal oriented error estimators for Hartree Fock and Kohn Sham equations (DFT)

Reinhold Schneider[1] S. Schwinger

Hartree Fock and DFT (density functional theory) is designed for computing the ground energy of a nonrelativistic N electron system, with various application in chemistry, solid physics and materialscience. Within the framework of *weighted dual residual methods* we present a posteriori error estimators measuring the error in the desired energy computation.

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Mixed finite elements for thin elastic structures

Astrid Sinwel[1] Joachim Schöberl[2]

In this talk, we are concerned with solving the equations of linear elasticity on an anisotropic, tensor product domain $\Omega = \Omega_x \times \Omega_z$. There, $\Omega_x \subset \mathbb{R}^2$ is supposed to be a connected, Lipschitzian domain, whereas $\Omega_z = (0, d_z)$ corresponds to the thickness direction.

We derive a mixed variational formulation for the equations of linear elasticity

$$\begin{aligned} -\mathrm{div}(\sigma) &= f, \\ A\sigma &= \varepsilon(u). \end{aligned}$$

Here u denotes the displacement field, and σ the symmetric stress tensor. The linearized strain tensor $\varepsilon(u) = \frac{1}{2}(\nabla u + \nabla u^T)$ is given by the symmetric part of the gradient.

We choose $u \in H(\mathrm{curl})$, which implies tangential continuity of the displacement field. Starting from shape-regular, simplicial meshes for Ω_x, Ω_z we discretize the displacement field using Nédélec finite elements on the resulting, anisotropic tensor product mesh. For the stresses σ, this implies the necessity to have symmetric, tensor valued finite elements, where the normal normal component $\sigma_{nn} = n^T \sigma n$ is continuous across interfaces. We already developed conforming finite elements for simplicial meshes. As an extension, we propose a family of such elements of arbitrary order on the tensor product mesh.

Our elements are suitable for the discretization of beams or shells, as they do not suffer from shear locking. We see that the optimal order error estimates for the solution still hold true in the case of thin structures. We can show that the error depends on the anisotropic mesh sizes h_x, h_z, but not on their ratio. This is supported by our numerical results.

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L^∞-error estimates on graded meshes and its application to optimal control

Dieter Sirch[1] Thomas Apel[2] Arnd Rösch[3]

In this talk we present an L^∞-error estimate of the finite element approximation of an elliptic boundary value problem with Dirichlet boundary conditions in domains with corners. To achieve an approximation rate of $h^2|\ln h|$ the mesh has to be appropriately graded near corners with an interior angle larger than $\frac{\pi}{2}$. In contrast to previous publications the norm of the function, that has to be approximated, is seperated from the constants in this estimate.

We apply this result to a linear-quadratic optimal control problem with constraints on the control. For discretizing the state linear finite elements are used, the control is approximated by using piecewise constant ansatz functions. Approximations of the optimal solution of the continuous optimal control problem are constructed by a projection of the discrete adjoint state. For this approximation a convergence rate of $h^2|\ln h|$ in the maximum norm is shown.

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On numerical solution of fluid-structure interaction problems: application on flow in a channel with moving walls

Petr Sváček[1]

In this paper the numerical solution of two dimensional fluid-structure interaction problems is addressed. We consider the fully coupled formulation of incompressible viscous fluid flow over a structure, which is described by a limited number of generalized coordinates.

The mathematical formulation of a relevant fluid-structure interaction problem is given. For the flow model we use the incompressible system of Navier-Stokes equations with large values of the Reynolds number $10^4 - 10^6$. The Navier-Stokes equations are spatially discretized by the FE method and stabilized with a modification of the Galerkin Least Squares (GLS) method; cf. [2].

The motion of the computational domain is treated with the aid of Arbitrary Lagrangian Eulerian(ALE) method. The GLS stabilizing terms are modified in a consistent way with the weak formulation of the ALE method.

The structure model is considered as a solid body with several degrees of freedom and described by a system of nonlinear ordinary differential equations. The construction of the ALE mapping is based on the solution of an elastic problem.

The method is applied onto a problem of interaction of channel flow with moving walls (a model of flow through a vocal fold).

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Space-time discretizations for semilinear evolution problems

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We deal with a numerical solution of a scalar non-stationary semilinear convection-diffusion equation, which represents a simplified model problem for the solution of the system of the compressible Navier-Stokes equations. The space semi-discretization is carried out with the aid of the discontinuous Galerkin finite element method (DGFEM), which is based on a piecewise discontinuous polynomial approximation.

The resulting system of the ordinary differential equations is often discretized by the explicit Runge-Kutta methods since these schemes have a high order of accuracy and they are simple for implementation. Their drawback is a strong restriction to the length of the time step. In order to avoid this disadvantage it is suitable to use an implicit time discretization, but a fully implicit scheme leads to a necessity to solve a nonlinear system of algebraic equations at each time step which is rather expensive.

Therefore, we develop a higher order unconditionally stable (or with a large domain of stability) time discretization technique which does not require a solution of nonlinear algebraic problem at each time step.

We analyse several approaches and derive a priori error estimates in the discrete analogues of the $L^\infty(0,T;L^2(\Omega))$-norm and the $L^2(0,T;H^1(\Omega))$-seminorm.

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New contributions to the free energy in the phase-field modeling of CVI

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Chemical vapor infiltration (CVI) is the most popular commercial method to produce the ceramic-matrix composite. In the CVI process for the production of silicon carbide (SiC), the SiC matrix is chemically deposited around the fibers during the complex pyrolysis of methyltrichlorosilane (MTS) [1]. The uniform densification and low residual porosity are two critical ascepts to get the high-performance ceramic-martix composites. The prediction of the blockage of the premature pore plays an important role in process optimisation. Thereforce we need a mathematical model to predict the evolution of the gas-solid-interface of this phase tranisition process. The sharp interface method has been used to describe the evolution of the gas-solid interface for a 1D-model [2]. However, for higher dimensions, the phase field approach seems more attractive and has been widely used [3].

The evolution of the gas-solid interface is implictly described by the evolution of the phase-field parameter ϕ. A Landau-Ginzburg type equation for the evolution of ϕ is deduced based on the free energy $\mathcal{F}(\phi)$. Here we propose new contributions to the free energy: first, to more realistically model the physical processes in the diffuse domain, we introduce the scalar-valued phase-field-dependent process intensities for the homogenous and heterogeneous process in the CVI process. Moreover, the potential energy part of the free energy is constructed by the restrictions on its local minima in the pure gas and in the pure solid phase. The whole phase-field system couples the nonlinear evolution equation for with the balance equation of the physical processes in CVI. Simulations of a 2D isothermal CVI process of SiC have been performed using the finite element method [4]. The new structure of the potential energy part has additionally been found to improve the numerical stability of the solution.

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Mumford-Shah-based elastic shape averaging

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A method is presented to compute an average representation of a given number of shapes. The method is based on image registration via edge matching and uses a hyperelastic regularization. The corresponding model is stated in a variational form and implemented as a fixed point iteration of gradient descent steps, using finite element methods in a multilevel framework. Results are presented that show the model's applicability to finding e. g. the average shape of a human organ or even to produce simple anatomical template maps.

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Boundary Element Methods for Dirichlet Boundary Control Problems

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For the solution of optimal Dirichlet boundary control problems we propose and analyze two different boundary element approaches. The state equation, the adjoint equation, and the optimality condition are rewritten as systems of boundary integral equations involving the standard boundary integral operators of the Laplace and of the Bi-Laplace equation. While the first approach is based on the use of the weakly singular Bi-Laplace boundary integral equation only, the additional use of the hypersingular Bi-Laplace boundary integral equation results in a symmetric formulation. We prove the unique solvability of both boundary integral approaches, and discuss related boundary element discretizations.

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Conforming hp finite elements for pyramids

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The efficient meshing of complicated geometries often relies on the simultaneous use of tensor-product and unstructured simplicial meshes. A conforming coupling requires also pyramidal elements. The construction of finite elements on pyramids is somewhat non-standard, leading e.g. to rational basis functions.

We present a general framework for the construction of conforming high-order pyramidal elements for $H^1(\Omega)$, $H(\mathrm{curl}, \Omega)$, $H(\mathrm{div}, \Omega)$, and $L_2(\Omega)$. Our approach relies on the explicit treatment of higher-order kernel and range spaces of the canonical differential operator. The constructed sequence of finite element spaces automatically inherits the global exactness of a continuous de Rham sequence, which provides stable discretizations for mixed problems as well as spectral correctness of $H(\mathrm{curl})$- and $H(\mathrm{div})$-conforming finite elements. Moreover, the construction of parameter-robust Schwarz-type preconditioners easily carries over to pyramidal elements.

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