Multidimensional Coupling in a Human Knee Model

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We present a new way to couple bones modelled as linear elastic three-dimensional objects to ligaments modelled as Cosserat rods. Starting from a full 3d nonlinear elastic formulation we derive suitable coupling conditions for the reduced model. These involve the total force and torque transmitted through the interface as well as its averaged position and an average orientation. The resulting domain decomposition problem is solved using a Dirichlet–Neumann algorithm.

The configuration space of a special Cosserat rod is the set of all continuous mappings from a given interval to $\mathbb{R}^3 \times \mathrm{SO}(3)$. We introduce geodesic finite elements as a natural way to discretize problems in such a nonlinear space. For the minimization of the rod energy functional we present an ∞-norm Riemannian trust-region algorithm. This yields an efficient method with provable global convergence.

We use this coupling approach to model a human knee joint. The use of rods for the ligaments decreases the overall number of degrees of freedom and avoids meshing problems. The additional problem of modelling the contact between the bones is treated using a mortar element discretization and a truncated nonsmooth Newton multigrid method for the solution of the resulting discrete system.

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