A two step time discretization of Willmore flow

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In surface restoration a damaged region of a surface is replaced by a surface patch which restores the region in a suitable way. In particular one aims for a $C1$-continuity at the patch boundary. The $L2$-gradient flow of the Willmore functional is considered to measure fairness and to allow appropriate boundary conditions ensuring continuity of the normal field. Often, real world restoration problems are of anisotropic nature, e.g., if an edge or a corner of a surface is destroyed. The generalization of the classical Willmore functional is a fourth order energy having Wulff shapes as the only minimizers. The corresponding $L2$-gradient flow as the actual restoration process leads to a system of fourth order partial differential equation. Usual discretizations approach for this PDE require the evaluation of higher order derivatives of the anistropy and come along with strong restrictions on the time step. To overcome these difficulties we develop a two step time discretization of the Willmore flow. It is based on abstract approaches for gradient flows which balance energy decay and dissipation encoded in the metric. In a variational approach for the time discrete Willmore flow we have to solve a nested variational problem in each time step. The inner minimization problem represents a time step of mean curvature motion on the unknown surface at the next time step. The discrete speed of propagation from this scheme is considered as a suitable approximation of the mean curvature vector. Hence, this vector is incorporated in the energy term of the outer minimization problem for a time step of Willmore flow. Due to the built-in regularized fully implicit Willmore functional the resulting scheme turned out to be very robust in numerical experiments and time steps of the order of the spatial grid size are feasible.

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