The one-level approach of the local projection method applied to inf-sup stable discretisations of the Oseen problem

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The Oseen problem occurs as an important subproblem during the iterative solution of the stationary and instationary Navier–Stokes equations.

The standard Galerkin finite element method can be applied after choosing approximation spaces for velocity and pressure. If these spaces satisfy an inf-sup condition, no pressure stabilisation is needed. However, the Galerkin method suffers in general from spurious oscillations in the velocity which are caused by the dominating convection.

To handle this instability, the local projection stabilisation will be used. Originally, the local projection technique was proposed as a two-level method where the projection space is defined on a coarser mesh. Unfortunately, this approach leads to an increased discretisation stencil.

Our main objective is to analyse the convergence properties of the one-level approach of the local projection stabilisation applied to inf-sup stable discretisations of the Oseen problem. Moreover, we propose new inf-sup stable finite element pairs approximating both velocity and pressure by elements of order r. In contrast to the 'classical' equal order interpolation, the velocity components and the pressure are discretised by different elements. In the convection dominated case $\nu < h$, we show for these pairs of finite element spaces an error estimate of order $r + 1/2$.

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