A rigorous analysis of FETI methods for high-contrast coefficients not resolved by the subdomain partitioning

Clemens Pechstein[1] Robert Scheichl[2]

Finite element tearing and interconnecting (FETI) methods are highly efficient parallel iterative substructuring solvers, i. e., non-overlapping domain decomposition methods. In the scalar elliptic case $-\nabla \cdot [\alpha \nabla u] = f$, it is known that the condition number of the preconditioned system is bounded by $C\,(1 + \log(H/h))^2$, where h is the mesh parameter, H is the subdomain diameter, and the constant C is independent of h, H, and the number of subdomains. In the case that the coefficient $\alpha(\cdot)$ is constant on each subdomain, the constant C in the estimate is independent of $\alpha(\cdot)$, i. e., the method is robust with respect to large jumps across subdomain interfaces. In the general case, however, a straightfoward estimate delivers $C \lesssim \max\limits_i \max\limits_{x,y\in\Omega_i} \frac{\alpha(x)}{\alpha(y)}$, where the Ω_i are the subdomains.

As observed numerically by several authors (e. g., [Rixen, Farhat, IJNME 44, 1999], [Klawonn, Rheinbach, CMAME 196, 2007], [Langer, Pechstein, ZAMM 86, 2006], simple generalizations of FETI-type method are still robust even in case of large coefficient variation inside the subdomains. In this talk we present a rigorous analysis for a class of non-trivial coefficient distributions. In particular we can allow for arbitrary coefficient values in the subdomain interiors, and two different materials with moderate coefficient variation in the remainder of each subdomain. Our bounds are independent of jump sizes but essentially depend on the geometry of the materials. In particular we can prove the robustness observed by Rixen and Farhat. The first author has been supported by the Austrian Science Founds (FWF) under grant F1306.

[1]Johannes Kepler Universität, Institut für Numerische Mathematik, Altenberger Str. 69, 4040 Linz, Austria,
clemens.pechtein@numa.uni-linz.ac.at
[2]Department of Mathematical Sciences, University of Bath, Bath BA2 7AY, United Kingdom,
r.scheichl@maths.bath.ac.uk