Solving elliptic PDEs in high space dimensions

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We consider the problem of finding $u \in H_0^1(0,1)^d$ such that $a(u,v) = f(v)$ for all $v \in H_0^1(0,1)^d$, where a is an elliptic bilinear form. Specifically, we try to do this for large space dimensions d. When the unknown solution u is approximated using standard isotropic approximation with piecewise polynomials of a fixed degree, we run into the so-called 'curse of dimensionality': the convergence rate is inversely proportional to d.

Using that $(0,1)^n$ is a tensor-product domain, the curse of dimensionality can be circumvented using a sparse tensor product approximation. However, this can only be expected to work when special regularity conditions are met. Already for the Poisson equation with constant (non-zero) right-hand side, this is not the case.

We use an adaptive wavelet approximation method, which reaches a convergence rate as that of the best N-term approximation, in linear complexity. For this, we use orthogonal tensor product wavelets based on the multiwavelets of Donovan et al.

Numerical results will be shown for experiments in high dimensions, illustrating the convergence rate.

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