A new Approach to Recovery of Discontinuous Galerkin

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A new recovery operator $P : Q_n^{disc}(\mathcal{T}) \to Q_{n+1}^{disc}(\mathcal{M})$ for discontinuous Galerkin is derived. It is based on the idea of projecting the local polynomial solution on a given mesh $\mathcal{T}$ into a higher order polynomial space on a macro mesh $\mathcal{M}$. Therefore, we define local degrees of freedom using Legendre polynomials and provide global degrees of freedom on the macro mesh.
We prove consistency

$$Pu = Pu^I, \forall u \in L_1(M)$$

with u^I as the local L_2-projection, stability results in several norms and optimal anisotropic error estimates.
In case of bilinear elements and the singularly perturbed problem

$$\begin{aligned} -\varepsilon\Delta u + \mathbf{b}\cdot\nabla u + cu &= f, \quad \text{in } \Omega = (0,1)\times(0,1) \\ u &= 0, \quad \text{on } \Gamma = \partial\Omega \end{aligned}$$

we use a supercloseness result by H.-G. Roos and H. Zarin to prove superconvergence of the postprocessed numerical solution.

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