Equilibrated residual-based error estimators for Poisson and Maxwell's equations

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Equilibrated a-posteriori error estimators for Poisson equations rely on the following principle: Any flux σ which fulfils the equilibrium condition, $div\,\sigma = f$, can be used to obtain an upper bound for the discretization error without generic constants. We present a patch-wise construction of a flux correction $\sigma^{\Delta} = \sigma + grad(u_h)$ that is also applicable for high-order FE-schemes and for Maxwell's equations.

The flux correction has to satisfy $div(\sigma^{\Delta}) = f + \Delta u_h$ in the distributional sense. The existence of a solution of the local vertex-patch problems is guaranteed by the exactness of discrete distributional de Rham sequences. The appropriate finite element spaces are a broken Raviart-Thomas space for the flux correction and an element-wise polynomial space extended by polynomials living only on the skeleton for the residual.

The extension to curl-curl problems follows an analogue principle. Here, the divergence-free localization of the residual to local patch-problems plays a crucial role.

We illustrate the advantages of our method by numerical experiments with h- and p-refinement strategies.

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