Higher order time discretizations for scalar nonlinear convection-diffusion equation

Miloslav Vlasak[1] Vit Dolejsi[2]

We deal with a numerical solution of a scalar non-stationary nonlinear convection-diffusion equation, which we use as a model simplified problem for the solution of the system of the compressible Navier-Stokes equations. For the space discretization we use the discontinuous Galerkin finite element method (DGFEM). We employ the so-called NIPG (nonsymmetric interior penalty Galerkin) approach. After discretization by DGFEM we obtain system of the ordinary differential equations (ODEs), which is often discretized by the explicit Runge-Kutta methods since these schemes have a high order of accuracy and they are simple for implementation. Their drawback is a strong restriction to the length of the time step. In order to avoid this disadvantage it is suitable to use an implicit time discretization, but a fully implicit scheme leads to a necessity to solve a nonlinear system of algebraic equations at each time step which is rather expensive and complicated. Therefore, we develop a higher order unconditionally stable (or with a large stability domain) time discretization technique which do not require a solution of nonlinear problem at each time step. We present several approaches and derive some a priori error estimates in the discrete analogous of the $L^\infty(0,T;L^2(\Omega))$-norm and the $L^2(0,T;H^1(\Omega))$-seminorm.

[1]Charles University in Prague, Faculty of Mathematics and Physics, Department of Numerical Mathematics, Sokolovska 83, 18675 Prague 8, Czech Republic,
vlasakmm@volny.cz

[2]Charles University in Prague, Faculty of Mathematics and Physics, Department of Numerical Mathematics, Sokolovska 83, 18675 Prague 8,
dolejsi@karlin.mff.cuni.cz