Piecewise Parabolic Method on Local stencil

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A new numerical method for solving of fluid dynamics and magnetic hydrodynamics equations is suggested. This method is a modification of popular Piecewise Parabolic Method (PPM) (P. Collela, P. Woodward, J. Comp. Phys.,**54**, 1984, 174.). PPM approved itself in different problems of computational physics, it has third-order approximation error for space variables and second-order for time variable. One of its disadvantages is the use of the interpolation procedure on four-point stencil for computation of quantities' values on the boundaries of the cells of difference grid to construct piecewise parabolas. This procedure leads to dissipation of quantities and to smoothing shock's fronts and contact discontinuities in Riemann problem for discontinuity decay. In our paper we suggested a way to avoid the interpolation procedure with the help of information from previous time step - we used the property of Riemann invariants to conserve when moving along the characteristic curves of hyperbolic system of equations. For the first time a similar idea was applied to construct a difference scheme with linear approximation of quantities inside cells long before PPM was invented (B. van Leer, J. Comp. Phys., **14**, 1974, 361.). We suggest the name of modified method as Piecewise Parabolic Method on Local stencil (PPML).

PPML was compared with PPM by numerical solution of Cauchy problem for linear advection equation and inviscid Burgers' equation. The comparison was carried out on the basis of values of errors in different norms and showed that PPML is more accurate. A numerical scheme based on PPML was developed for fluid dynamics and MHD problems solution. Much attention was paid to a monotonicity preserving problem. A question about maintaining of nondivergence constraint on magnetic field was also considered in MHD.

We made a number of standard tests: Sod, Lax, Shu etc problems for 1D hydrodynamics; 2D Riemann problem with contact discontinuities, the double-Mach reflection problem in 2D hydrodynamics; Woodward and Colella's interacting-blast-wave problem, 2.5D shock tube problem, Rotor problem, Orszag-Tang vortex problem etc in MHD. The method was successfully applied to study 3D isothermal MHD turbulence, which is rather difficult problem because of strong shock-waves and regions of high rarefaction.

PPML approved itself as accurate, low-dissipative and convenient to use in adaptive grids.

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