An Unconditionally Stable Mixed Discontinuous Galerkin Method

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In discontinuous Galerkin method continuity can be forced in various ways. One quite popular way to force continuity in discontinuous approach is the Nitsche method. The method is based on handling of non-homogenous Dirichlet boundary conditions by J.A. Nitsche (1970). However, for non-mixed problems the method is elliptic (or stable) only for large enough values of stability parameter. Lower limit of the stability parameter depends on the degrees of freedom, e.g. on polynomial order of the elements. Even though the lower limit can be computed analytically, it may cause problems in for example p-refinement (i.e. refinement by increasing the polynomial order of the elements). Therefore it is interesting to notice that we can give up the stability parameter if we pose the problem in a mixed form.

In this work we show that for mixed problem the Nitsche method is stable for all values of the stability parameter, hence we can give up the parameter entirely. We also propose a residual based a posteriori error estimate for the method. The proof the a posteriori estimate is based on the Helmholtz decomposition, in which the gradient is decomposed into divergence free and rotation free parts. The Helmholtz decomposition technique is also applicable in other similar proofs. The advantage of the Helmholtz decomposition in the discontinuous framework is that we can still use the conventional interpolants, for example Clment's interpolants, since the members of the decomposition are continuous and we only need the interpolants for the decomposition.

Our model problem is the mixed form of the Poisson equation, in other words, the gradient of the solution is also a variable. Solving mixed equations naturally increases size of the problem. However, we can reduce the increase of the computational work conciderably with local condensation. We show that it is possible to solve the gradient part of the mixed solution already from the local, elementwise, matrix equation. Thus the size of the ultimate matrix equation is the same as usual, only the assembly is more toilsome since every element requires solving a small matrix equation.

In addition to the analytical results we show numerical examples supporting the results.

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