Numerical Analysis of Stokes Equations with Improved LBB Dependency

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We provide a priori bounds with improved domain dependency both for the solution of Stokes Equations and for the numerical error of an approximation by conforming Finite Element Methods. The domain dependency mainly appears in terms of the inverse of the LBB constant $\frac{1}{L}$. It is known from several previous works, that L decreases with the aspect ratio of the domain. In the first part we explain the LBB dependency of common a priori bounds on Du and p. Then, we improve most of these estimates by avoiding a global inf-sup condition and by the assumption of *locally-balanced flow*.
In fact, most common Finite Element Methods do not show a dependency of the aspect ratio at all. Hence, in a last step we explain how these methods, which we characterize as *LBB-friendly*, reduce the LBB dependency by automatically subtracting an appropriate base flow. For certain domains, if this modification is carried out manually, at least the condition of locally balanced flow can be roughly achieved. But if the same procedure is accomplished automatically by the choice of an appropriate method, locally-balanced flow can even be assumed for the dual problem and hence improve the L^2-pressure error estimate. Finally, we present some numerical computations which demonstrate the sharpness of the theoretical results.

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