Mixed fem-bem coupling for non-linear transmission problems with Signorini contact

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Here we generalize the approach in [4] and discuss an interface problem consisting of a non-linear partial differential equation in $\Omega \subset \mathbb{R}^n$ (bounded, Lipschitz, $n \geq 2$) and the Laplace equation in the unbounded exterior domain $\Omega_c := \mathbb{R}^n \backslash \bar{\Omega}$ fulfilling some radiation condition, which are coupled by transmission conditions and Signorini conditions imposed on the interface. The interior pde is discretized by a mixed formulation, whereas the exterior part of the interface problem is rewritten using a Neumann to Dirichlet mapping (NtD) given in terms of boundary integral operators.

We treat the general numerical approximation of the resulting variational inequality and discuss the non-trivial discretization of the NtD mapping. Assuming some abstract approximation properties and a discrete inf-sup condition we prove existence and uniqueness and show an a-priori estimate, which generalizes the results in [4]. Choosing Raviart-Thomas elements and piecewise constants in Ω and hat functions on $\partial\Omega$ the discrete inf-sup condition is satisfied [1]. We present a solver based on a modified Uzawa algorithm, reducing the solution procedure of the non-linear saddle point problem with an inequality constraint to the repeated solution of a standard non-linear saddle point problem and the solution of a variational inequality based on an elliptic operator. Finally, we present a residual based a-posteriori error estimator compatible with the Signorini condition and a corresponding adaptive scheme, see [5].

Some numerical experiments are shown which illustrate the convergence behavior of the uniform h-version with triangles and rectangles and the adaptive scheme as well as the bounded iteration numbers of the modified Uzawa algorithm, underlining the theoretical results.

REFERENCES

[1] I. Babuška, G. Gatica "On the mixed finite element method with Lagrange multipliers", Numer. Methods Partial Differential Equations, Vol. 19, pp. 192–210, (2003).

[2] M. Barrientos, G. Gatica, M. Maischak "A-posteriori error estimates for linear exterior problems via mixed-FEM and DtN mappings", Mathematical Modelling and Numerical Analysis, Vol. 36, pp. 241–272 (2002).

[3] G. Gatica, M. Maischak "A posteriori error estimates for the mixed finite element method with Lagrange multipliers", Numerical Methods for Partial Differential Equations, Vol. 21, pp. 421–450, (2005).

[4] G. Gatica, M. Maischak, E.P. Stephan "Mixed FEM and BEM coupling for a transmission problem with Signorini contact", NMPDE (submitted)

[5] M. Maischak "A posteriori error estimates for a mixed fem-bem coupling method with Signorini contact", (submitted).

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