Stabilization by local projection. Convection-diffusion and incompressible flow problems

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The discretisation of the Oseen problem by finite element methods may suffer in general from two shortcomings. First, the discrete inf-sup (Babuška–Brezzi) condition can be violated. Second, spurious oscillations occur due to the dominating convection. One way to overcome both difficulties is the use of local projection techniques. Studying the local projection method in an abstract setting, we show that the fulfilment of a local inf-sup condition between approximation and projection spaces allows to construct an interpolation with additional orthogonality properties. Based on this special interpolation, optimal a-priori error estimates are shown with error constants independent of the Reynolds number. Applying the general theory, we extend the results of Braack and Burman for the standard two-level version of the local projection stabilisation to discretisations of arbitrary order on simplices, quadrilaterals, and hexahedra. Moreover, our general theory allows to derive a novel class of local projection stabilisation by enrichment of the approximation spaces. This class of stabilised schemes uses approximation and projection spaces defined on the same mesh and leads to much more compact stencils than in the two-level approach. Finally, on simplices, the spectral equivalence of the stabilising terms of the local projection method and the subgrid modeling introduced by Guermond is shown. This clarifies the relation of the local projection stabilisation to the variational multiscale approach.

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