On discontinuous Galerkin methods for convection-dominated elliptic problems

Herbert Egger[1] Joachim Schöberl[2]

Convection dominated elliptic problems cannot be solved on a reasonable discretization level by standard conforming Galerkin FEM. Moreover, stabilization by upwinding techniques like SUPG is known to introduce significant artificial diffusion, which has certain drawbacks, e.g., the widening of boundary layers.

For this reason, discontinuous Galerkin (DG) methods, which are well-known for their good performance for the limiting hyperbolic problem, have been applied also to elliptic problems. A major disadvantage of DG methods is that the discretization of the elliptic operator leads to increased stencils, i.e., the resulting system matrices are much less sparse then the ones coming from conforming methods.

We propose a new method for convection-diffusion problems, which is composed of a mixed method for the elliptic operator and a discontinuous Galerkin discretization of the convective part. This method has the stability and conservation property of DG methods while leading to much sparser systems. Moreover, we outline the convergence analysis and present numerical tests.

[1]RWTH Aachen, Center for Computational Engineering Science (CCES), Pauwelstrasse 19, 52074 Aachen, Germany,
herbert.egger@rwth-aachen.de
[2]RWTH Aachen,