Sparse Second Moment Analysis for Potentials on Stochastic Domains

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This talk is concerned with the numerical solution of Dirichlet problems in domains $D \in \mathbb{R}^d$ with random boundary perturbations. Assuming normal perturbations with small amplitude and known mean field and two-point correlation function, we derive, using a second order shape calculus, deterministic equations for the mean field and the two-point correlation function of the random solution for the Dirichlet problem in the stochastic domain.

The two-point correlation of the random solution satisfies a boundary value problem on the tensor product domain $D \times D$. It can be approximated in sparse tensor product spaces. This yields densely populated system matrices, independently of using the finite element method in $D \times D$ or the boundary element method on $\partial D \times \partial D$.

We present and analyze algorithms to approximate the random solution's two-point correlation function in essentially $\mathcal{O}(N)$ work and memory, where N denotes the number of unknowns required for consistent discretization of the domain (in case of finite element methods) or its boundary (in case of boundary element methods). Here "essentially" means up to powers of $\log N$.

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