Reduced Basis Method for Evolution Schemes with Nonlinear Explicit Operators

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During the last decades, reduced basis (RB) methods have been developed to a wide methodology for model reduction of problems that are governed by parametrized partial differential equations [1]. In particular equations of elliptic and parabolic type for linear, low polynomial or monotonic nonlinearities have been treated successfully by RB methods using finite element schemes. Due to the characteristic offline-online decomposition, the reduced models often become suitable for a multi-query or real-time setting, where simulation results, such as field-variables or output estimates, can be approximated reliably and rapidly for varying parameters.

In the current study, we address a certain class of time-dependent nonlinear evolution schemes. We extend the linear scheme [2] with a general nonlinear explicit space-discretization operator. We extend the RB-methodology to these cases by applying the *empirical interpolation* method [3] to localized discretization operators. The main technical ingredients are: (i) generation of a *collateral reduced basis* modelling the nonlinearity under parameter variations in the offline-phase and (ii) an online simulation scheme based on localized evaluations of the evolution operator.

Numerical experiments on a parametrized convection-diffusion problem, discretized with a finite volume scheme, demonstrate the applicability of the model reduction technique. We obtain a parametrized reduced model, which enables parameter variation with fast simulation response. We quantify the computational gain with respect to the non-reduced model and investigate the error convergence.

[1] A.T. Patera and G. Rozza. *Reduced Basis Approximation and A Posteriori Error Estimation for Parametrized Partial Differential Equations.* Version 1.0, Copyright MIT 2006-2007, to appear in (tentative rubric) MIT Pappalardo Graduate Monographs in Mechanical Engineering.

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