On discrete projection methods for rotating incompressible flow with Coriolis force

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In many physical and industrial processes there is a necessity of numerical simulations of models with moving boundaries in a 3-dimensional space. As proposed in the literature, approaches like fictitious boundary or Arbitrary Lagrangian Eulerian methods require large amount of CPU time to simulate even a 2D benchmark models. Moreover, they could provide a source of additional errors in velocity and pressure fields. At the same time, there is a large class of rotating models, where the use of the above methods can be avoided by some modifications of Navier-Stokes equations and/or transformations of the coordinate system. In our talk we present a numerical analysis and algorithmic details for treating the system of Stokes and Navier-Stokes equations with Coriolis force term. Using a Discrete Projection Method with modified Pressure Schur Complement operator, we examine the influence of the Coriolis force on every step of the algorithm: modification of the momentum equation and building of a new block diagonal preconditioner, construction of a new Pressure Schur Complement operator by inserting the off-diagonal parts, which are due to the rotating terms and an explicit formation of its inverse, and finally, correction of pressure and velocity to satisfy the incompressibility constraints. Detailed numerical studies of the improvements in the convergence rate, its dependence on the magnitude of the time step and angular velocity are provided. We also consider the addition of the convection term into the Schur Complement, discuss its influence on the iterative behaviour of the solver and propose a generalized coriolis-convection oriented Pressure Schur Complement. Finally, we provide numerical simulations for polygonal geometries and check the efficiency of our solver by comparing simulation results with those obtained by the fictitious boundary method for moving boundary parts. The used solver is based on the 3-dimensional PP3D CFD-code from the Featflow package (www.featflow.de).

References:

[1] S. Turek. On discrete projection methods for the incompressible Navier-Stokes equations: an algorithmical approach, Comput. Methods Appl. Mech. Engrg., Vol. 143, page 271-288, 1997.
[2] M. A. Olshanskii. An iterative Solver for the Oseen Problem and Numerical Solution of Incompressible Navier-Stokes Equations, Numer. Linear Algebra Appl., Vol. 6, page 353-378, 1999
[3] S. Turek. Efficient Solvers for Incompressible Flow Problems: An Algorithmic and Computational Approach, LNCSE 6, Springer, 1999.
[4] S. Turek, D. C. Wan and L. P. Rivkind. The Fictitious Boundary Method for the implicit treatment of Dirichlet boundary conditions with applications to incompressible flow simulations, In E. Bänsch editor, Challenges in Scientific Computing CISC 2002, LNCSE, page 37-68, Berlin, 2002. Springer, Berlin.

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