Non-Nested Multi-Grid Solvers for Mixed Divergence-free Scott-Vogelius Discretizations

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In [1] a general framework for analyzing the convergence of multi-level methods for mixed finite element problems has been established. Important applications for this framework are nearly all practically relevant conforming and non-conforming mixed finite element discretizations of the Stokes problem. When applying this framework to discretizations of the Navier-Stokes equation, additional difficulties have to be taken into account, e.g. the grad-div stabilization for stabilizing weak mass conservation.

In the talk, we present basic results for a recently proposed conforming stabilized discretization of the full Navier-Stokes problem [2]. The discretization is based on the Scott-Vogelius element, and uses symmetric stabilization operators for stabilizing dominant convection. Here, discrete approximations to the Navier-Stokes problem are pointwise divergence-free. Problems induced by the cumbersome grad-div stabilization are therefore circumvented. For this discretization, we apply the general multi-grid framework [1] and analyze a generalized Stokes problem including the symmetric stabilization operator for dominant convection. Since the Scott-Vogelius element has only been proven to be LBB-stable on meshes, which are derived from a macroelement triangulation, the multigrid hierarchy is non-nested.

References:

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