A high-order non-conforming finite element family

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We describe a triangular non-conforming finite element pair for the two-dimensional Stokes problem [6]. It is a generalization of the low order cases described by Crouzeix and Raviart [3], by Fortin and Soulie [4] and by Crouzeix and Falk [2]. The velocity and the pressure are approximated trianglewise by polynomials of order k and k-1, respectively, and the element is defined for all k$\geq$1. For even k the finite element pair can be obtained by adding trianglewise a non-conforming bubble function of order k to the local basis of the velocity space of the conforming $\mathbb{P}_k/\mathbb{P}_{k-1}$ element, and we give a general formula for these bubble functions. In the case of odd k we describe a set of degrees of freedom for the velocity part of the element. For even k using a modification of the macroelement technique of Stenberg [5] we prove that the finite element pair is inf-sup stable [1].

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