Hardy space infinite elements for scattering and resonance problems

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We study the solution of time-harmonic wave equations in unbounded domains. The unbounded domain is split into a bounded interior domain and an exterior domain, which is the complement of a ball. We propose a new class of tensor product infinite elements for the exterior domain which lead to super-algebraic convergence with respect to the number of degrees of freedom. The radial tensor product factors of the local element matrices have a simple tridiagonal structure.

To derive these infinite elements, we use a Möbius-Laplace transform along a family of rays connecting the coupling boundary to infinity. By virtue of the pole condition, functions satisfying a radiation condition are mapped to Hardy-space functions, i.e. L^2-boundary values of holomorphic functions on the unit disc. This leads to a complex symmetric variational formulation, which is discretized by a Galerkin method using trigonometric polynomials of finite degree.

Hardy space infinite elements are particularly attractive for computing resonances since, as opposed to classical infinite elements, boundary elements and local transparent boundary conditions they preserve the eigenvalue structure of the problem. We demonstrate their performance in a number of numerical experiments.

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