Non-conforming finite elements of arbitrary order for the Stokes problem on anisotropic meshes

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Anisotropic meshes are characterized by elements with large or even asymptotically unbounded aspect ratio. Such meshes are known to be particularly effective for the resolution of directional features of the solution, like edge singularities and boundary layers.

We consider here the numerical solution of the Stokes problem in two-dimensional domains by non-conforming finite elements of higher order. The pressure is approximated by discontinuous, piecewise polynomials of order $r-1$. For approximating the velocity we discuss four non-conforming spaces of approximation order r.

For the stability of finite element methods for solving the Stokes problem it is necessary that the discrete spaces fulfil an inf-sup condition. All of the considered families fulfill this condition but only two of them have an inf-sup constant which is independent of the aspect ratio of the meshes. For these two families we show optimal error estimates on anisotropic meshes. The proof is restricted to rectangular triangulations with special properties.

References:

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