Anisotropic mesh adaption based on a posteriori estimates and optimisation of node positions

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Efficient numerical approximation of solution features like boundary or interior layers by means of the finite element method requires the use of layer adapted meshes. Anisotropic meshes, like for example Shishkin meshes, allow the most efficient approximation of these highly anisotropic solution features. However, application of this approach relies on empha priori analysis on the thickness, position and stretching direction of the layers. If it is impossible to obtain this information empha priori, as it is often the case for problems with interior layers of unknown position for example, automatic mesh adaption based on empha posteriori error estimates or error indicators is essential in order to obtain efficient numerical approximations.

Historically the majority of work on automatic mesh adaption used locally uniform refinement, splitting each element into smaller elements of similar shape. This procedure is clearly not suitable to produce anisotropically refined meshes. The resulting meshes are over-refined in at least one spatial direction, rendering the approach far less efficient than that of the anisotropic meshes based on empha priori analysis.

Automatic anisotropic mesh adaption is an area of active research. Here we present a new approach to this problem, based upon using not only an empha posteriori error estimate to guide the mesh refinement, but its sensitivities with respect to the positions of the nodes in the mesh as well. Once this sensitivity information is available, techniques from mathematical optimisation can be used to minimise the estimated error by moving the positions of the nodes in the mesh appropriately.

The basic idea of minimising an error estimate is of course not new, but the approach taken to realise it is. The discrete adjoint technique is utilised to evaluate the sensitivities of an error estimate, reducing the cost of this evaluation to solving one additional equation system. This approach is crucial to make gradient based optimisation techniques, such as BFGS-type schemes, applicable.

References:

[1] `http://www.tu-chemnitz.de/~rens/`

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